

Solving Linear Equations

What You'll Learn

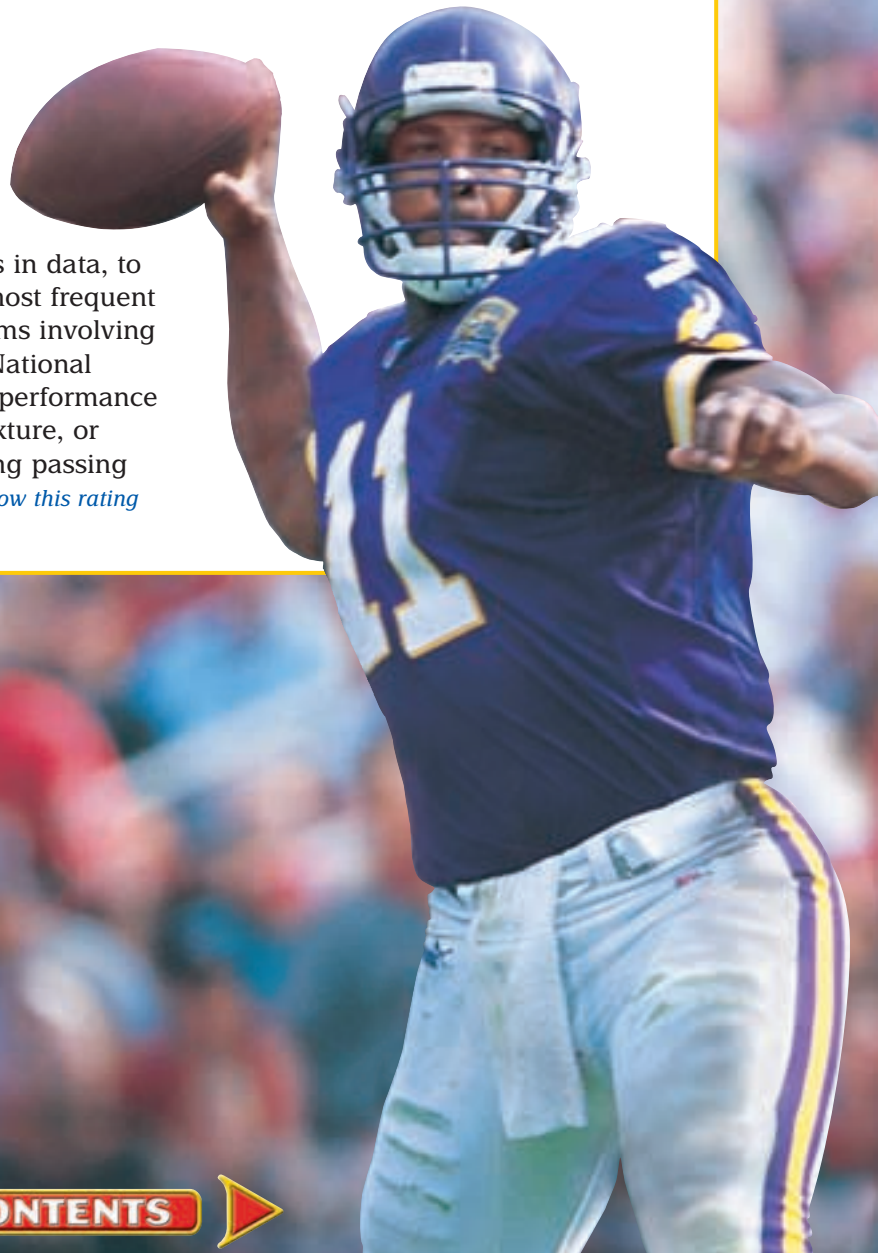
- **Lesson 3-1** Translate verbal sentences into equations and equations into verbal sentences.
- **Lessons 3-2 through 3-6** Solve equations and proportions.
- **Lesson 3-7** Find percents of change.
- **Lesson 3-8** Solve equations for given variables.
- **Lesson 3-9** Solve mixture and uniform motion problems.

Key Vocabulary

- equivalent equations (p. 129)
- identity (p. 150)
- proportion (p. 155)
- percent of change (p. 160)
- mixture problem (p. 171)

Why It's Important

Linear equations can be used to solve problems in every facet of life from planning a garden, to investigating trends in data, to making wise career choices. One of the most frequent uses of linear equations is solving problems involving mixtures or motion. For example, in the National Football League, a quarterback's passing performance is rated using an equation based on a mixture, or weighted average, of five factors, including passing attempts and completions. *You will learn how this rating system works in Lesson 3-9.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 3.

For Lesson 3-1

Write Mathematical Expressions

Write an algebraic expression for each verbal expression. (For review, see Lesson 1-1.)

- five greater than half of a number t
- the product of seven and s divided by the product of eight and y
- the sum of three times a and the square of b
- w to the fifth power decreased by 37
- nine times y subtracted from 95
- the sum of r and six divided by twelve

For Lesson 3-4

Use the Order of Operations

Evaluate each expression. (For review, see Lesson 1-2.)

- $3 \cdot 6 - \frac{12}{4}$
- $5(13 - 7) - 22$
- $5(7 - 2) - 3^2$
- $\frac{2 \cdot 6 - 4}{2}$
- $(25 - 4) \div (2^2 - 1)$
- $36 \div 4 - 2 + 3$
- $\frac{19 - 5}{7} + 3$
- $\frac{1}{4}(24) - \frac{1}{2}(12)$

For Lesson 3-7

Find the Percent

Find each percent. (For review, see pages 802 and 803.)

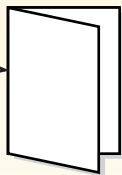
- Five is what percent of 20?
- What percent of 5 is 15?
- Sixteen is what percent of 10?
- What percent of 300 is 21?
- Twelve is what percent of 60?
- What percent of 50 is 37.5?

FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about solving linear equations. Begin with 4 sheets of plain $8\frac{1}{2}$ " by 11" paper.

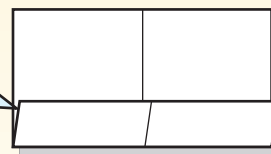
Step 1 Fold

Fold in half along the width.



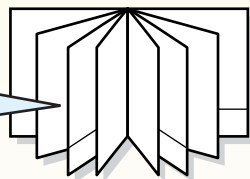
Step 2 Open and Fold Again

Fold the bottom to form a pocket. Glue edges.



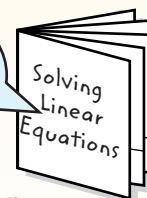
Step 3 Repeat Steps 1 and 2

Repeat three times and glue all four pieces together.



Step 4 Label

Label each pocket. Place an index card in each pocket.



Reading and Writing As you read and study the chapter, you can write notes and examples on each index card.

3-1 Writing Equations

What You'll Learn

- Translate verbal sentences into equations.
- Translate equations into verbal sentences.

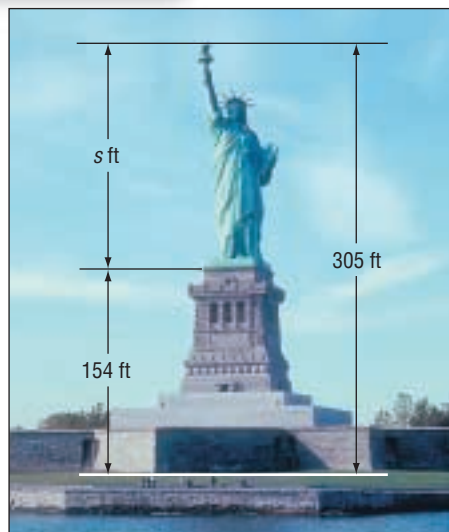
Vocabulary

- four-step problem-solving plan
- defining a variable
- formula

How are equations used to describe heights?

The Statue of Liberty sits on a pedestal that is 154 feet high. The height of the pedestal and the statue is 305 feet. If s represents the height of the statue, then the following equation represents the situation.

$$154 + s = 305$$



Source: World Book Encyclopedia

WRITE EQUATIONS When writing equations, use variables to represent the unspecified numbers or measures referred to in the sentence or problem. Then write the verbal expressions as algebraic expressions. Some verbal expressions that suggest the *equals sign* are listed below.

- is
- is equal to
- is as much as
- equals
- is the same as
- is identical to

Example 1 Translate Sentences into Equations

Translate each sentence into an equation.

- a. Five times the number a is equal to three times the sum of b and c .

$$\underbrace{\text{Five}}_5 \underbrace{\text{times}} \times \underbrace{a} = \underbrace{\text{is equal to}} = \underbrace{\text{three}}_3 \underbrace{\text{times}} \times \underbrace{\text{the sum of } b \text{ and } c}_{(b + c)}$$

The equation is $5a = 3(b + c)$.

- b. Nine times y subtracted from 95 equals 37.

Rewrite the sentence so it is easier to translate.

$$\underbrace{95} \underbrace{\text{less}} - \underbrace{\text{nine times } y}_{9y} \underbrace{\text{equals}} = \underbrace{37}$$

The equation is $95 - 9y = 37$.

Study Tip

Look Back

To review translating verbal expressions to algebraic expressions, see Lesson 1-1.

Using the **four-step problem-solving plan** can help you solve any word problem.

Key Concept

Four-Step Problem-Solving Plan

- Step 1** Explore the problem.
- Step 2** Plan the solution.
- Step 3** Solve the problem.
- Step 4** Examine the solution.

Each step of the plan is important.

Study Tip

Reading Math

In a verbal problem, the sentence that tells what you are asked to find usually contains *find*, *what*, *when*, or *how*.

Step 1 Explore the Problem

To solve a verbal problem, first read the problem carefully and explore what the problem is about.

- Identify what information is given.
- Identify what you are asked to find.

Step 2 Plan the Solution

One strategy you can use to solve a problem is to write an equation. Choose a variable to represent one of the unspecified numbers in the problem. This is called **defining a variable**. Then use the variable to write expressions for the other unspecified numbers in the problem. *You will learn to use other strategies throughout this book.*

Step 3 Solve the Problem

Use the strategy you chose in Step 2 to solve the problem.

Step 4 Examine the Solution

Check your answer in the context of the original problem.

- Does your answer make sense?
- Does it fit the information in the problem?

More About . . .



Ice Cream

The first ice cream plant was established in 1851 by Jacob Fussell. Today, 2,000,000 gallons of ice cream are produced in the United States each day.

Source: *World Book Encyclopedia*

Example 2 Use the Four-Step Plan

- **ICE CREAM** Use the information at the left. In how many days can 40,000,000 gallons of ice cream be produced in the United States?

Explore You know that 2,000,000 gallons of ice cream are produced in the United States each day. You want to know how many days it will take to produce 40,000,000 gallons of ice cream.

Plan Write an equation to represent the situation. Let d represent the number of days needed to produce the ice cream.

$$\underbrace{2,000,000}_{\text{2,000,000}} \times \underbrace{\text{times}}_{\times} \underbrace{\text{the number of days}}_d \text{ equals } \underbrace{40,000,000}_{40,000,000}$$

Solve $2,000,000d = 40,000,000$ Find d mentally by asking, "What number times 2,000,000 equals 40,000,000?"
 $d = 20$

It will take 20 days to produce 40,000,000 gallons of ice cream.

Examine If 2,000,000 gallons of ice cream are produced in one day, $2,000,000 \times 20$ or 40,000,000 gallons are produced in 20 days. The answer makes sense.



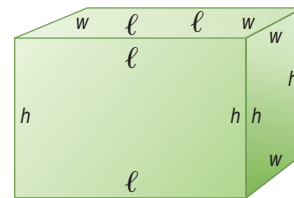
A **formula** is an equation that states a rule for the relationship between certain quantities. Sometimes you can develop a formula by making a model.



Algebra Activity

Surface Area

- Mark each side of a rectangular box as the length ℓ , the width w , or the height h .
- Use scissors to cut the box so that each surface or face of the box is a separate piece.



Analyze

1. Write an expression for the area of the front of the box.
2. Write an expression for the area of the back of the box.
3. Write an expression for the area of one side of the box.
4. Write an expression for the area of the other side of the box.
5. Write an expression for the area of the top of the box.
6. Write an expression for the area of the bottom of the box.
7. The surface area of a rectangular box is the sum of all the areas of the faces of the box. If S represents surface area, write a formula for the surface area of a rectangular box.

Make a Conjecture

8. If s represents the length of the side of a cube, write a formula for the surface area of a cube.

Example 3 Write a Formula

Translate the sentence into a formula.

The perimeter of a rectangle equals two times the length plus two times the width.



Words Perimeter equals two times the length plus two times the width.

Variables Let P = perimeter, ℓ = length, and w = width.

Formula

<u>Perimeter</u>	<u>equals</u>	<u>two times the length</u>	<u>plus</u>	<u>two times the width.</u>
P	$=$	2ℓ	$+$	$2w$

The formula for the perimeter of a rectangle is $P = 2\ell + 2w$.

WRITE VERBAL SENTENCES You can also translate equations into verbal sentences or make up your own verbal problem if you are given an equation.

Example 4 Translate Equations into Sentences

Translate each equation into a verbal sentence.

a. $3m + 5 = 14$

$\underbrace{3m}_{\text{Three times } m} + \underbrace{5}_{\text{plus five}} = \underbrace{14}_{\text{equals fourteen}}$

Study Tip

Look Back
To review translating algebraic expressions to verbal expressions, see Lesson 1-1.



b. $w + v = y^2$

The sum of w and v equals the square of y .

Example 5 Write a Problem

Write a problem based on the given information.

$a = \text{Rafael's age}$ $a + 5 = \text{Tierra's age}$ $a + 2(a + 5) = 46$

You know that a represents Rafael's age and $a + 5$ represents Tierra's age. The equation adds a plus twice $(a + 5)$ to get 46. A sample problem is given below.

Tierra is 5 years older than Rafael. The sum of Rafael's age and twice Tierra's age equals 46. How old is Rafael?

Check for Understanding

Concept Check

- List the four steps used in solving problems.
- Analyze the following problem.

Misae has \$1900 in the bank. She wishes to increase her account to a total of \$3500 by depositing \$30 per week from her paycheck. Will she reach her savings goal in one year?

- How much money did Misae have in her account at the beginning?
 - How much money will Misae add to her account in 10 weeks? in 20 weeks?
 - Write an expression representing the amount added to the account after w weeks have passed.
 - What is the answer to the question? Explain.
- OPEN ENDED** Write a problem that can be answered by solving $x + 16 = 30$.

Guided Practice

Translate each sentence into an equation.

- Two times a number t decreased by eight equals seventy.
- Five times the sum of m and n is the same as seven times n .

Translate each sentence into a formula.

- The area A of a triangle equals one half times the base b times the height h .
- The circumference C of a circle equals the product of two, pi, and the radius r .

Translate each equation into a verbal sentence.

- $14 + d = 6d$
- $\frac{1}{3}b - \frac{3}{4} = 2a$
- Write a problem based on the given information.
 $c = \text{cost of a suit}$ $c - 25 = 150$

Application

WRESTLING For Exercises 11 and 12, use the following information.

Darius is training to prepare for wrestling season. He weighs 155 pounds now. He wants to gain weight so that he starts the season weighing 160 pounds.

- If g represents the number of pounds he wants to gain, write an equation to represent the situation.
- How many pounds does Darius need to gain to reach his goal?



Practice and Apply

Homework Help

For Exercises	See Examples
13–22	1
23–28	3
29–38	4
39, 40	5
41–51	2

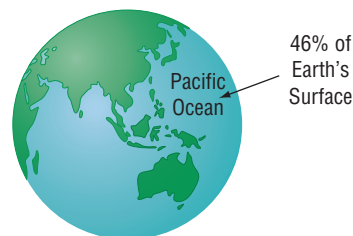
Extra Practice

See page 825.

Translate each sentence into an equation.

- Two hundred minus three times x is equal to nine.
- The sum of twice r and three times s is identical to thirteen.
- The sum of one-third q and 25 is as much as twice q .
- The square of m minus the cube of n is sixteen.
- Two times the sum of v and w is equal to two times z .
- Half of the sum of nine and p is the same as p minus three.
- The number g divided by the number h is the same as seven more than twice the sum of g and h .
- Five-ninths the square of the sum of a , b , and c equals the sum of the square of a and the square of c .

- GEOGRAPHY** The Pacific Ocean covers about 46% of Earth. If P represents the area of the Pacific Ocean and E represents the area of Earth, write an equation for this situation.

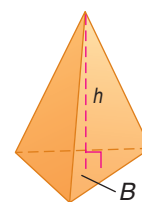
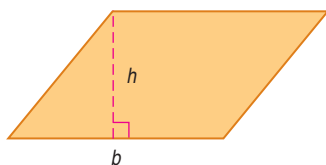


Source: World Book Encyclopedia

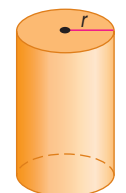
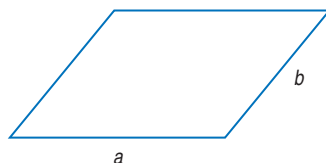
- GARDENING** Mrs. Patton is planning to place a fence around her vegetable garden. The fencing costs \$1.75 per yard. She buys f yards of fencing and pays \$3.50 in tax. If the total cost of the fencing is \$73.50, write an equation to represent the situation.

Translate each sentence into a formula.

- The area A of a parallelogram is the base b times the height h .
- The volume V of a pyramid is one-third times the product of the area of the base B and its height h .



- The perimeter P of a parallelogram is twice the sum of the lengths of the two adjacent sides, a and b .
- The volume V of a cylinder equals the product of π , the square of the radius r of the base, and the height.



- In a right triangle, the square of the measure of the hypotenuse c is equal to the sum of the squares of the measures of the legs, a and b .
- The temperature in degrees Fahrenheit F is the same as nine-fifths of the degrees Celsius C plus thirty-two.

Translate each equation into a verbal sentence.

29. $d - 14 = 5$

30. $2f + 6 = 19$

31. $k^2 + 17 = 53 - j$

32. $2a = 7a - b$

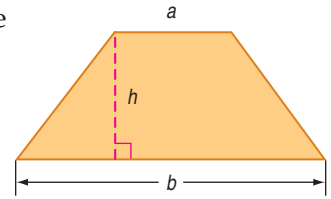
33. $\frac{3}{4}p + \frac{1}{2} = p$

34. $\frac{2}{5}w = \frac{1}{2}w + 3$

35. $7(m + n) = 10n + 17$

36. $4(t - s) = 5s + 12$

37. **GEOMETRY** If a and b represent the lengths of the bases of a trapezoid and h represents its height, then the formula for the area A of the trapezoid is $A = \frac{1}{2}h(a + b)$. Write the formula in words.



38. **SCIENCE** If r represents rate, t represents time, and d represents distance, then $rt = d$. Write the formula in words.

Write a problem based on the given information.

39. y = Yolanda's height in inches
 $y + 7$ = Lindsey's height in inches
 $2y + (y + 7) = 193$

40. p = price of a new backpack
 $0.055p$ = tax
 $p + 0.055p = 31.65$

GEOMETRY For Exercises 41 and 42, use the following information.

The volume V of a cone equals one-third times the product of π , the square of the radius r of the base, and the height h .

41. Write the formula for the volume of a cone.
 42. Find the volume of a cone if r is 10 centimeters and h is 30 centimeters.

GEOMETRY For Exercises 43 and 44, use the following information.

The volume V of a sphere is four-thirds times π times the radius r of the sphere cubed.

43. Write a formula for the volume of a sphere.
 44. Find the volume of a sphere if r is 4 inches.

LITERATURE For Exercises 45–47, use the following information.

Edgar Rice Burroughs is the author of the *Tarzan of the Apes* stories. He published his first Tarzan story in 1912. Some years later, the town in southern California where he lived was named Tarzana.

45. Let y represent the number of years after 1912 that the town was named Tarzana. Write an expression for the year the town was named.
 46. The town was named in 1928. Write an equation to represent the situation.
 47. Use what you know about numbers to determine the number of years between the first Tarzan story and the naming of the town.

TELEVISION For Exercises 48–51, use the following information.

During a highly rated one-hour television program, the entertainment portion lasted 15 minutes longer than 4 times the advertising portion.

48. If a represents the time spent on advertising, write an expression for the entertainment portion.
 49. Write an equation to represent the situation.
 50. Use your equation and the guess-and-check strategy to determine the number of minutes spent on advertising. Choose different values of a and evaluate to find the solution.
 51. Time the entertainment and advertising portions of a one-hour television program you like to watch. Describe what you found. Are the results of this problem similar to your findings?

More About . . .



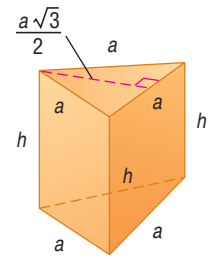
Literature

More than 50 movies featuring Tarzan have been made. The first, *Tarzan of the Apes*, in 1918, was among the first movies to gross over \$1 million.

Source: www.tarzan.org



52. **CRITICAL THINKING** The surface area of a prism is the sum of the areas of the faces of the prism. Write a formula for the surface area of the triangular prism at the right.



53. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are equations used to describe heights?

Include the following in your answer:

- an equation relating the Sears Tower, which is 1454 feet tall; the twin antenna towers on top of the building, which are a feet tall; and a total height, which is 1707 feet, and
- an equation representing the height of a building of your choice.

Standardized Test Practice

A B C D

54. Which equation represents the following sentence?
One fourth of a number plus five equals the number minus seven.
- (A) $\frac{1}{4}n + 7 = n - 5$ (B) $\frac{1}{4}n + 5 = n - 7$
(C) $4n + 7 = n - 5$ (D) $4n + 5 = n - 7$
55. Which sentence can be represented by $7(x + y) = 35$?
- (A) Seven times x plus y equals 35.
(B) One seventh of the sum of x and y equals 35.
(C) Seven plus x and y equals 35.
(D) Seven times the sum of x and y equals 35.

Maintain Your Skills

Mixed Review

Find each square root. Use a calculator if necessary. Round to the nearest hundredth if the result is not a whole number or a simple fraction. (Lesson 2-7)

56. $\sqrt{8100}$

57. $-\sqrt{\frac{25}{36}}$

58. $\sqrt{90}$

59. $-\sqrt{55}$

Find the probability of each outcome if a die is rolled. (Lesson 2-6)

60. a 6

61. an even number

62. a number greater than 2

Simplify each expression. (Lesson 1-5)

63. $12d + 3 - 4d$

64. $7t^2 + t + 8t$

65. $3(a + 2b) + 5a$

Evaluate each expression. (Lesson 1-2)

66. $5(8 - 3) + 7 \cdot 2$

67. $6(4^3 + 2^2)$

68. $7(0.2 + 0.5) - 0.6$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each sum or difference.

(To review operations with fractions, see pages 798 and 799.)

69. $5.67 + 3.7$

70. $0.57 + 2.8$

71. $5.28 - 3.4$

72. $9 - 7.35$

73. $\frac{2}{3} + \frac{1}{5}$

74. $\frac{1}{6} + \frac{2}{3}$

75. $\frac{7}{9} - \frac{2}{3}$

76. $\frac{3}{4} - \frac{1}{6}$



Algebra Activity

A Preview of Lesson 3-2

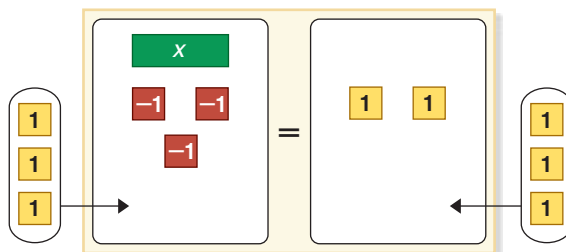
Solving Addition and Subtraction Equations

You can use algebra tiles to solve equations. To solve an equation means to find the value of the variable that makes the equation true. After you model the equation, the goal is to get the x tile by itself on one side of the mat using the rules stated below.

Rules for Equation Models	
You can remove or add the same number of identical algebra tiles to each side of the mat without changing the equation.	
One positive tile and one negative tile of the same unit are a zero pair . Since $1 + (-1) = 0$, you can remove or add zero pairs to the equation mat without changing the equation.	

Use an equation model to solve $x - 3 = 2$.

Step 1 Model the equation.

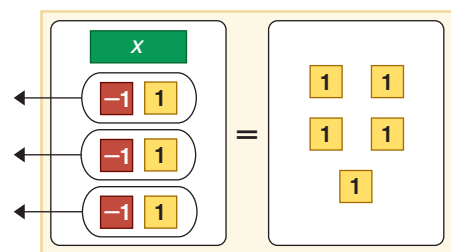


$$x - 3 = 2$$

$$x - 3 + 3 = 2 + 3$$

Place 1 x tile and 3 negative 1 tiles on one side of the mat. Place 2 positive 1 tiles on the other side of the mat. Then add 3 positive 1 tiles to each side.

Step 2 Isolate the x term.



$$x = 5$$

Group the tiles to form zero pairs. Then remove all the zero pairs. The resulting equation is $x = 5$.

Model and Analyze

Use algebra tiles to solve each equation.

1. $x + 5 = 7$

2. $x + (-2) = 28$

3. $x + 4 = 27$

4. $x + (-3) = 4$

5. $x + 3 = -4$

6. $x + 7 = 2$

Make a Conjecture

7. If $a = b$, what can you say about $a + c$ and $b + c$?

8. If $a = b$, what can you say about $a - c$ and $b - c$?

Solving Equations by Using Addition and Subtraction

What You'll Learn

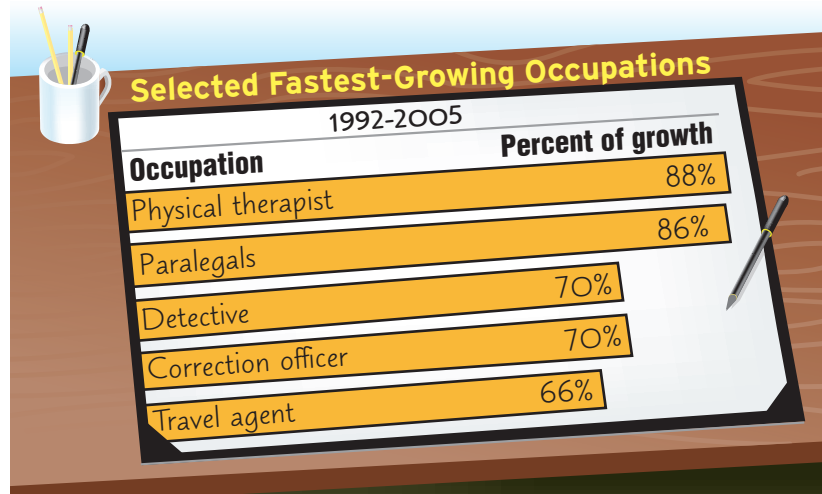
- Solve equations by using addition.
- Solve equations by using subtraction.

Vocabulary

- equivalent equation
- solve an equation

How can equations be used to compare data?

The graph shows some of the fastest-growing occupations from 1992 to 2005.



Source: Bureau of Labor Statistics

The difference between the percent of growth for medical assistants and the percent of growth for travel agents in these years is 5%. An equation can be used to find the percent of growth expected for medical assistants. If m is the percent of growth for medical assistants, then $m - 66 = 5$. You can use a property of equality to find the value of m .

SOLVE USING ADDITION Suppose your school's boys' soccer team has 15 members and the girls' soccer team has 15 members. If each team adds 3 new players, the number of members on the boys' and girls' teams would still be equal.

$$15 = 15 \quad \text{Each team has 15 members before adding the new players.}$$

$$15 + 3 = 15 + 3 \quad \text{Each team adds 3 new members.}$$

$$18 = 18 \quad \text{Each team has 18 members after adding the new members.}$$

This example illustrates the **Addition Property of Equality**.

Key Concept

Addition Property of Equality

• **Words** If the same number is added to each side of an equation, the resulting equation is true.

• **Symbols** For any numbers a , b , and c , if $a = b$, then $a + c = b + c$.

• **Examples**

$$7 = 7$$

$$14 = 14$$

$$7 + 3 = 7 + 3$$

$$14 + (-6) = 14 + (-6)$$

$$10 = 10$$

$$8 = 8$$

If the same number is added to each side of an equation, then the result is an equivalent equation. **Equivalent equations** have the same solution.

$$t + 3 = 5 \quad \text{The solution of this equation is 2.}$$

$$t + 3 + 2 = 5 + 2 \quad \text{Using the Addition Property of Equality, add 2 to each side.}$$

$$t + 5 = 7 \quad \text{The solution of this equation is also 2.}$$

Study Tip

Reading Math

Remember that x means $1 \cdot x$. The coefficient of x is 1.

To **solve an equation** means to find all values of the variable that make the equation a true statement. One way to do this is to isolate the variable having a coefficient of 1 on one side of the equation. You can sometimes do this by using the Addition Property of Equality.

Example 1 Solve by Adding a Positive Number

Solve $m - 48 = 29$. Then check your solution.

$$m - 48 = 29 \quad \text{Original equation}$$

$$m - 48 + 48 = 29 + 48 \quad \text{Add 48 to each side.}$$

$$m = 77 \quad -48 + 48 = 0 \text{ and } 29 + 48 = 77$$

To check that 77 is the solution, substitute 77 for m in the original equation.

CHECK $m - 48 = 29$ Original equation

$$77 - 48 \stackrel{?}{=} 29 \quad \text{Substitute 77 for } m.$$

$$29 = 29 \checkmark \quad \text{Subtract.}$$

The solution is 77.

Example 2 Solve by Adding a Negative Number

Solve $21 + q = -18$. Then check your solution.

$$21 + q = -18 \quad \text{Original equation}$$

$$21 + q + (-21) = -18 + (-21) \quad \text{Add } -21 \text{ to each side.}$$

$$q = -39 \quad 21 + (-21) = 0 \text{ and } -18 + (-21) = -39$$

CHECK $21 + q = -18$ Original equation

$$21 + (-39) \stackrel{?}{=} -18 \quad \text{Substitute } -39 \text{ for } q.$$

$$-18 = -18 \checkmark \quad \text{Add.}$$

The solution is -39 .

SOLVE USING SUBTRACTION Similar to the Addition Property of Equality, there is a **Subtraction Property of Equality** that may be used to solve equations.

Key Concept

Subtraction Property of Equality

- **Words** If the same number is subtracted from each side of an equation, the resulting equation is true.
- **Symbols** For any numbers a , b , and c , if $a = b$, then $a - c = b - c$.
- **Examples**

$17 = 17$	$3 = 3$
$17 - 9 = 17 - 9$	$3 - 8 = 3 - 8$
$8 = 8$	$-5 = -5$



Example 3 Solve by Subtracting

Solve $142 + d = 97$. Then check your solution.

$$142 + d = 97 \quad \text{Original equation}$$

$$142 + d - 142 = 97 - 142 \quad \text{Subtract 142 from each side.}$$

$$d = -45 \quad 142 - 142 = 0 \text{ and } 97 - 142 = -45$$

CHECK $142 + d = 97$ Original equation

$$142 + (-45) \stackrel{?}{=} 97 \quad \text{Substitute } -45 \text{ for } d.$$

$$97 = 97 \checkmark \quad \text{Add.}$$

The solution is -45 .

Remember that subtracting a number is the same as adding its inverse.

Example 4 Solve by Adding or Subtracting

Solve $g + \frac{3}{4} = -\frac{1}{8}$ in two ways.

Method 1 Use the Subtraction Property of Equality.

$$g + \frac{3}{4} = -\frac{1}{8} \quad \text{Original equation}$$

$$g + \frac{3}{4} - \frac{3}{4} = -\frac{1}{8} - \frac{3}{4} \quad \text{Subtract } \frac{3}{4} \text{ from each side.}$$

$$g = -\frac{7}{8} \quad \frac{3}{4} - \frac{3}{4} = 0 \text{ and } -\frac{1}{8} - \frac{3}{4} = -\frac{1}{8} - \frac{6}{8} \text{ or } -\frac{7}{8}$$

The solution is $-\frac{7}{8}$.

Method 2 Use the Addition Property of Equality.

$$g + \frac{3}{4} = -\frac{1}{8} \quad \text{Original equation}$$

$$g + \frac{3}{4} + \left(-\frac{3}{4}\right) = -\frac{1}{8} + \left(-\frac{3}{4}\right) \quad \text{Add } -\frac{3}{4} \text{ to each side.}$$

$$g = -\frac{7}{8} \quad \frac{3}{4} + \left(-\frac{3}{4}\right) = 0 \text{ and } -\frac{1}{8} + \left(-\frac{3}{4}\right) = -\frac{1}{8} + \left(-\frac{6}{8}\right) \text{ or } -\frac{7}{8}$$

The solution is $-\frac{7}{8}$.

Example 5 Write and Solve an Equation

Write an equation for the problem. Then solve the equation and check your solution.

A number increased by 5 is equal to 42. Find the number.

$$\underbrace{\text{A number}}_n \quad \underbrace{\text{increased by}}_+ \quad \underbrace{5}_5 \quad \underbrace{\text{is equal to}}_= \quad \underbrace{42}_{42}$$

$$n + 5 = 42 \quad \text{Original equation}$$

$$n + 5 - 5 = 42 - 5 \quad \text{Subtract 5 from each side.}$$

$$n = 37 \quad 5 - 5 = 0 \text{ and } 42 - 5 = 37$$

CHECK $n + 5 = 42$ Original equation

$$37 + 5 \stackrel{?}{=} 42 \quad \text{Substitute 37 for } n.$$

$$42 = 42 \checkmark$$

The solution is 37.

Study Tip

Checking Solutions

You should always check your solution in the context of the original problem. For instance, in Example 5, is 37 increased by 5 equal to 42? The solution checks.



History

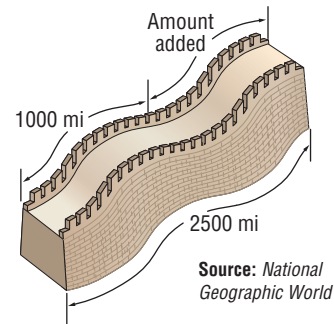
The first emperor of China, Qui Shi Huangdi, ordered the building of the Great Wall of China to protect his people from nomadic tribes that attacked and looted villages. By 204 B.C., this wall guarded 1000 miles of China's border.

Source: National Geographic World

Example 6 Write an Equation to Solve a Problem

HISTORY Refer to the information at the right.

In the fourteenth century, the part of the Great Wall of China that was built during Qui Shi Huangdi's time was repaired, and the wall was extended. When the wall was completed, it was 2500 miles long. How much of the wall was added during the 1300s?



Source: National Geographic World

Words The original length plus the additional length equals 2500.

Variable Let a = the additional length.

Equation

The original length	plus	the additional length	equals	2500.
1000	+	a	=	2500

$1000 + a = 2500$	Original equation
$1000 + a - 1000 = 2500 - 1000$	Subtract 1000 from each side.
$a = 1500$	$1000 - 1000 = 0$ and $2500 - 1000 = 1500.$

The Great Wall of China was extended 1500 miles in the 1300s.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write three equations that are equivalent to $n + 14 = 27$.
2. **Compare and contrast** the Addition Property of Equality and the Subtraction Property of Equality.
3. **Show** two ways to solve $g + 94 = 75$.

Guided Practice

Solve each equation. Then check your solution.

- | | | |
|-------------------|----------------------|-------------------------------------|
| 4. $t - 4 = -7$ | 5. $p + 19 = 6$ | 6. $15 + r = 71$ |
| 7. $104 = y - 67$ | 8. $h - 0.78 = 2.65$ | 9. $\frac{2}{3} + w = 1\frac{1}{2}$ |

Write an equation for each problem. Then solve the equation and check your solution.

10. Twenty-one subtracted from a number is -8 . Find the number.
11. A number increased by -37 is -91 . Find the number.

Application

CARS For Exercises 12–14, use the following information.

The average time it takes to manufacture a car in the United States is equal to the average time it takes to manufacture a car in Japan plus 8.1 hours. The average time it takes to manufacture a car in the United States is 24.9 hours.

12. Write an addition equation to represent the situation.
13. What is the average time to manufacture a car in Japan?
14. The average time it takes to manufacture a car in Europe is 35.5 hours. What is the difference between the average time it takes to manufacture a car in Europe and the average time it takes to manufacture a car in Japan?



Practice and Apply

Homework Help

For Exercises	See Examples
15–40	1–4
41–48	5
51–64	6

Extra Practice

See page 825.

Solve each equation. Then check your solution.

15. $v - 9 = 14$

16. $s - 19 = -34$

17. $g + 5 = 33$

18. $18 + z = 44$

19. $a - 55 = -17$

20. $t - 72 = -44$

21. $-18 = -61 + d$

22. $-25 = -150 + q$

23. $r - (-19) = -77$

24. $b - (-65) = 15$

25. $18 - (-f) = 91$

26. $125 - (-u) = 88$

27. $-2.56 + c = 0.89$

28. $k + 0.6 = -3.84$

29. $-6 = m + (-3.42)$

30. $6.2 = -4.83 + y$

31. $t - 8.5 = 7.15$

32. $q - 2.78 = 4.2$

33. $x - \frac{3}{4} = \frac{5}{6}$

34. $a - \frac{3}{5} = -\frac{7}{10}$

35. $-\frac{1}{2} + p = \frac{5}{8}$

36. $\frac{2}{3} + r = -\frac{4}{9}$

37. $\frac{2}{3} = v + \frac{4}{5}$

38. $\frac{2}{5} = w + \frac{3}{4}$

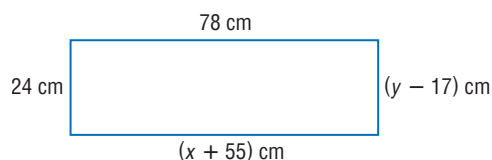
39. If $x - 7 = 14$, what is the value of $x - 2$?

40. If $t + 8 = -12$, what is the value of $t + 1$?

GEOMETRY For Exercises 41 and 42, use the rectangle at the right.

41. Write an equation you could use to solve for x and then solve for x .

42. Write an equation you could use to solve for y and then solve for y .



Write an equation for each problem. Then solve the equation and check your solution.

43. Eighteen subtracted from a number equals 31. Find the number.

44. What number decreased by 77 equals -18 ?

45. A number increased by -16 is -21 . Find the number.

46. The sum of a number and -43 is 102. What is the number?

47. What number minus one-half is equal to negative three-fourths?

48. The sum of 19 and 42 and a number is equal to 87. What is the number?

49. Determine whether $x + x = x$ is *sometimes*, *always*, or *never* true. Explain your reasoning.

50. Determine whether $x + 0 = x$ is *sometimes*, *always*, or *never* true. Explain your reasoning.

GAS MILEAGE For Exercises 51–55, use the following information.

A midsize car with a 4-cylinder engine goes 10 miles more on a gallon of gasoline than a luxury car with an 8-cylinder engine. A midsize car consumes one gallon of gas for every 34 miles driven.

51. Write an addition equation to represent the situation.

52. How many miles does a luxury car travel on a gallon of gasoline?

53. A subcompact car with a 3-cylinder engine goes 13 miles more than a luxury car on one gallon of gas. How far does a subcompact car travel on a gallon of gasoline?

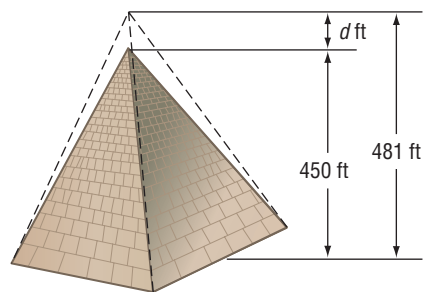
54. How many more miles does a subcompact travel on a gallon of gasoline than a midsize car?

55. Estimate how many miles a full-size car with a 6-cylinder engine goes on one gallon of gasoline. Explain your reasoning.

HISTORY For Exercises 56 and 57, use the following information.

Over the years, the height of the Great Pyramid at Giza, Egypt, has decreased.

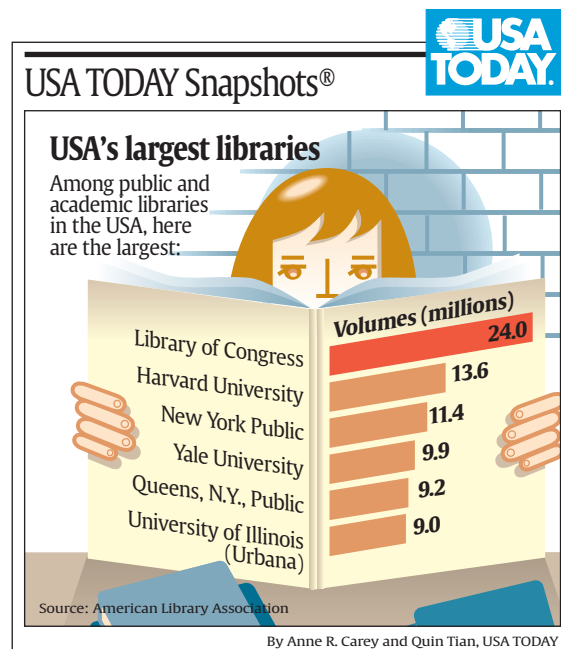
56. Write an addition equation to represent the situation.
57. What was the decrease in the height of the pyramid?



Source: World Book Encyclopedia

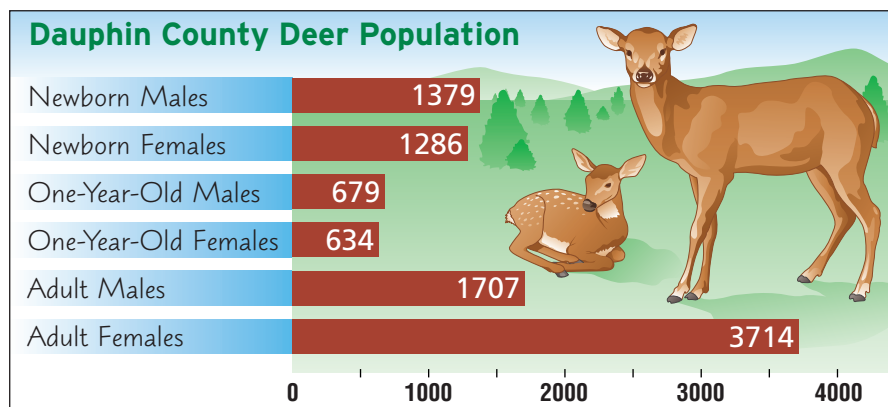
LIBRARIES For Exercises 58–61, use the graph at the right to write an equation for each situation. Then solve the equation.

58. How many more volumes does the Library of Congress have than the Harvard University Library?
59. How many more volumes does the Harvard University Library have than the New York Public Library?
60. How many more volumes does the Library of Congress have than the New York Public Library?
61. What is the total number of volumes in the three largest U.S. libraries?



ANIMALS For Exercises 62–64, use the information below to write an equation for each situation. Then solve the equation.

Wildlife authorities monitor the population of animals in various regions. One year's deer population in Dauphin County, Pennsylvania, is shown in the graph below.



Source: www.visi.com

62. How many more newborns are there than one-year-olds?
63. How many more females are there than males?
64. What is the total deer population?



65. **CRITICAL THINKING** If $a - b = x$, what values of a , b , and x would make the equation $a + x = b + x$ true?
66. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can equations be used to compare data?

Include the following in your answer:

- an explanation of how to solve the equation to find the growth rate for medical assistants, and
- a sample problem and related equation using the information in the graph.



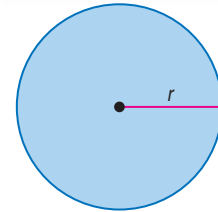
67. Which equation is *not* equivalent to $b - 15 = 32$?
- (A) $b + 5 = 52$ (B) $b - 20 = 27$
(C) $b - 13 = 30$ (D) $b = 47$
68. What is the solution of $x - 167 = -52$?
- (A) 115 (B) -115
(C) 219 (D) -219

Maintain Your Skills

Mixed Review

GEOMETRY For Exercises 69 and 70, use the following information.

The area of a circle is the product of π times the radius r squared. (Lesson 3-1)



69. Write the formula for the area of the circle.
70. If a circle has a radius of 16 inches, find its area.

Replace each \bullet with $>$, $<$, or $=$ to make the sentence true. (Lesson 2-7)

71. $\frac{1}{2} \bullet \sqrt{2}$

72. $\frac{3}{4} \bullet \frac{2}{3}$

73. $0.375 \bullet \frac{3}{8}$

Use each set of data to make a stem-and-leaf plot. (Lesson 2-5)

74. 54, 52, 43, 41, 40, 36, 35, 31, 32, 34, 42, 56
75. 2.3, 1.4, 1.7, 1.2, 2.6, 0.8, 0.5, 2.8, 4.1, 2.9, 4.5, 1.1

Identify the hypothesis and conclusion of each statement. (Lesson 1-7)

76. For $y = 2$, $4y - 6 = 2$.
77. There is a science quiz every Friday.

Evaluate each expression. Name the property used in each step. (Lesson 1-4)

78. $4(16 \div 4^2)$

79. $(2^5 - 5^2) + (4^2 - 2^4)$

Find the solution set for each inequality, given the replacement set. (Lesson 1-3)

80. $3x + 2 > 2$; $\{0, 1, 2\}$

81. $2y^2 - 1 > 0$; $\{1, 3, 5\}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each product or quotient.

(To review operations with fractions, see pages 800 and 801.)

82. 6.5×2.8 83. 70.3×0.15 84. $17.8 \div 2.5$ 85. $0.33 \div 1.5$
86. $\frac{2}{3} \times \frac{5}{8}$ 87. $\frac{5}{9} \times \frac{3}{10}$ 88. $\frac{1}{2} \div \frac{2}{5}$ 89. $\frac{8}{9} \div \frac{4}{15}$

Solving Equations by Using Multiplication and Division

What You'll Learn

- Solve equations by using multiplication.
- Solve equations by using division.

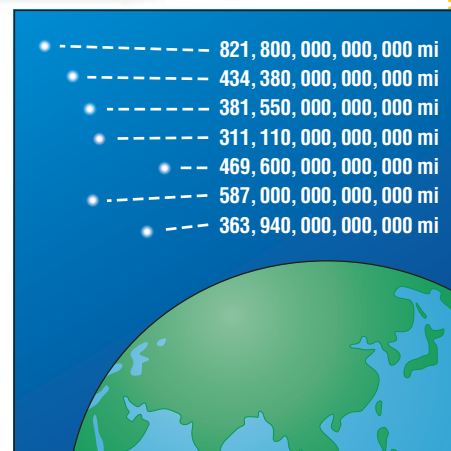
How can equations be used to find how long it takes light to reach Earth?

It may look like all seven stars in the Big Dipper are the same distance from Earth, but in fact, they are not. The diagram shows the distance between each star and Earth.

Light travels at a rate of about 5,870,000,000,000 miles per year. In general, the rate at which something travels times the time equals the distance ($rt = d$). The following equation can be used to find the time it takes light to reach Earth from the closest star in the Big Dipper.

$$rt = d$$

$$5,870,000,000,000t = 311,110,000,000,000$$



Source: National Geographic World

SOLVE USING MULTIPLICATION To solve equations such as the one above, you can use the **Multiplication Property of Equality**.

Key Concept

Multiplication Property of Equality

- **Words** If each side of an equation is multiplied by the same number, the resulting equation is true.
- **Symbols** For any numbers a , b , and c , if $a = b$, then $ac = bc$.
- **Examples**

$6 = 6$	$9 = 9$	$10 = 10$
$6 \times 2 = 6 \times 2$	$9 \times (-3) = 9 \times (-3)$	$10 \times \frac{1}{2} = 10 \times \frac{1}{2}$
$12 = 12$	$-27 = -27$	$5 = 5$

Example 1 Solve Using Multiplication by a Positive Number

Solve $\frac{t}{30} = \frac{7}{10}$. Then check your solution.

$$\frac{t}{30} = \frac{7}{10} \quad \text{Original equation}$$

$$30\left(\frac{t}{30}\right) = 30\left(\frac{7}{10}\right) \quad \text{Multiply each side by 30.}$$

$$t = 21 \quad \frac{t}{30}(30) = t \text{ and } \frac{7}{10}(30) = 21$$

(continued on the next page)

CHECK $\frac{t}{30} = \frac{7}{10}$ Original equation
 $\frac{21}{30} \stackrel{?}{=} \frac{7}{10}$ Substitute 21 for t .
 $\frac{7}{10} = \frac{7}{10}$ ✓ The solution is 21.

Example 2 Solve Using Multiplication by a Fraction

Solve $(2\frac{1}{4})g = 1\frac{1}{2}$.

$(2\frac{1}{4})g = 1\frac{1}{2}$ Original equation

$(\frac{9}{4})g = \frac{3}{2}$ Rewrite each mixed number as an improper fraction.

$\frac{4}{9}(\frac{9}{4})g = \frac{4}{9}(\frac{3}{2})$ Multiply each side by $\frac{4}{9}$, the reciprocal of $\frac{9}{4}$.

$g = \frac{12}{18}$ or $\frac{2}{3}$ Check this result.

The solution is $\frac{2}{3}$.

Example 3 Solve Using Multiplication by a Negative Number

Solve $42 = -6m$.

$42 = -6m$ Original equation

$-\frac{1}{6}(42) = -\frac{1}{6}(-6m)$ Multiply each side by $-\frac{1}{6}$, the reciprocal of -6 .

$-7 = m$ Check this result.

The solution is -7 .

You can write an equation to represent a real-world problem. Then use the equation to solve the problem.

Example 4 Write and Solve an Equation Using Multiplication

SPACE TRAVEL Refer to the information about space travel at the left. The weight of anything on the moon is about one-sixth its weight on Earth. What was the weight of Neil Armstrong's suit and life-support backpacks on Earth?

Words One sixth times the weight on Earth equals the weight on the moon.

Variable Let w = the weight on Earth.

Equation $\frac{\text{One sixth}}{\frac{1}{6}} \cdot \frac{\text{times}}{\cdot} \frac{\text{the weight on Earth}}{w} = \frac{\text{equals}}{=} \frac{\text{the weight on the moon.}}{33}$

$\frac{1}{6}w = 33$ Original equation

$6(\frac{1}{6}w) = 6(33)$ Multiply each side by 6.

$w = 198$ $\frac{1}{6}(6) = 1$ and $33(6) = 198$

The weight of Neil Armstrong's suit and life-support backpacks on Earth was about 198 pounds.



More About . . .

Space Travel

On July 20, 1969, Neil Armstrong stepped on the surface of the moon. On the moon, his suit and life-support backpacks weighed about 33 pounds.

Source: NASA

SOLVE USING DIVISION The equation in Example 3, $42 = -6m$, was solved by multiplying each side by $-\frac{1}{6}$. The same result could have been obtained by dividing each side by -6 . This method uses the **Division Property of Equality**.

Key Concept

Division Property of Equality

- **Words** If each side of an equation is divided by the same nonzero number, the resulting equation is true.
- **Symbols** For any numbers a , b , and c , with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
- **Examples**

$$15 = 15 \quad 28 = 28$$

$$\frac{15}{3} = \frac{15}{3} \quad \frac{28}{-7} = \frac{28}{-7}$$

$$5 = 5 \quad -4 = -4$$

Example 5 Solve Using Division by a Positive Number

Solve $13s = 195$. Then check your solution.

$$13s = 195 \quad \text{Original equation}$$

$$\frac{13s}{13} = \frac{195}{13} \quad \text{Divide each side by 13.}$$

$$s = 15 \quad \frac{13s}{13} = s \text{ and } \frac{195}{13} = 15$$

CHECK $13s = 195$ Original equation

$$13(15) \stackrel{?}{=} 195 \quad \text{Substitute 15 for } s.$$

$$195 = 195 \quad \checkmark$$

The solution is 15.

Study Tip

Alternative Method

You can also solve equations like those in Examples 5, 6, and 7 by using the Multiplication Property of Equality. For instance, in Example 6, you could multiply each side by $-\frac{1}{3}$.

Example 6 Solve Using Division by a Negative Number

Solve $-3x = 12$.

$$-3x = 12 \quad \text{Original equation}$$

$$\frac{-3x}{-3} = \frac{12}{-3} \quad \text{Divide each side by } -3.$$

$$x = -4 \quad \frac{-3x}{-3} = x \text{ and } \frac{12}{-3} = -4$$

The solution is -4 .

Example 7 Write and Solve an Equation Using Division

Write an equation for the problem below. Then solve the equation.
Negative eighteen times a number equals -198 .

$$\underbrace{\text{Negative eighteen}}_{-18} \quad \underbrace{\text{times}}_{\times} \quad \underbrace{\text{a number}}_n \quad \underbrace{\text{equals}}_{=} \quad \underbrace{-198}_{-198}$$

$$-18n = -198 \quad \text{Original equation}$$

$$\frac{-18n}{-18} = \frac{-198}{-18} \quad \text{Divide each side by } -18.$$

$$n = 11 \quad \text{Check this result.}$$

The solution is 11.



Check for Understanding

- Concept Check**
- OPEN ENDED** Write a multiplication equation that has a solution of -3 .
 - Explain** why the Multiplication Property of Equality and the Division Property of Equality can be considered the same property.
 - FIND THE ERROR** Casey and Juanita are solving $8n = -72$.

Casey

$$8n = -72$$

$$8n(8) = -72(8)$$

$$n = -576$$

Juanita

$$8n = -72$$

$$\frac{8n}{8} = \frac{-72}{8}$$

$$n = -9$$

Who is correct? Explain your reasoning.

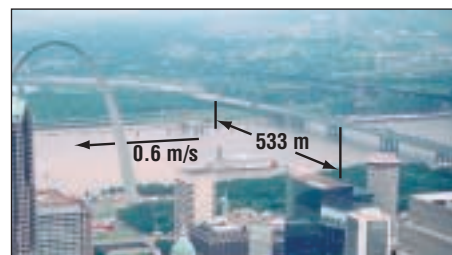
Guided Practice Solve each equation. Then check your solution.

- $-2g = -84$
- $\frac{a}{36} = \frac{4}{9}$
- $3.15 = 1.5y$
- $\frac{t}{7} = -5$
- $\frac{4}{5}k = \frac{8}{9}$
- $(3\frac{1}{4})p = 2\frac{1}{2}$

Write an equation for each problem. Then solve the equation.

- Five times a number is 120. What is the number?
- Two fifths of a number equals -24 . Find the number.

- Application**
- GEOGRAPHY** The discharge of a river is defined as the width of the river times the average depth of the river times the speed of the river. At one location in St. Louis, the Mississippi River is 533 meters wide, its speed is 0.6 meter per second, and its discharge is 3198 cubic meters per second. How deep is the Mississippi River at this location?



Practice and Apply

Homework Help

For Exercises	See Examples
13–32	1–3, 5, 6
33–38	7
39–49	4

Extra Practice
See page 826.

Solve each equation. Then check your solution.

- $-5r = 55$
 - $-1634 = 86s$
 - $\frac{2}{3}n = 14$
 - $\frac{z}{45} = \frac{2}{5}$
 - $-2.8m = 9.8$
 - $(3\frac{2}{3})x = -5\frac{1}{2}$
 - $8d = 48$
 - $\frac{b}{7} = -11$
 - $\frac{2}{5}g = -14$
 - $1.9f = -11.78$
 - $-5.73q = 97.41$
 - $-5h = -3\frac{2}{3}$
 - $-910 = -26a$
 - $-\frac{v}{5} = -45$
 - $\frac{g}{24} = \frac{5}{12}$
 - $0.49k = 6.272$
 - $(-2\frac{3}{5})t = -22$
 - $3p = 4\frac{1}{5}$
- If $4m = 10$, what is the value of $12m$?
 - If $15b = 55$, what is the value of $3b$?

Write an equation for each problem. Then solve the equation.

33. Seven times a number equals -84 . What is the number?
34. Negative nine times a number is -117 . Find the number.
35. One fifth of a number is 12. Find the number.
36. Negative three eighths times a number equals 12. What is the number?
37. Two and one half times a number equals one and one fifth. Find the number.
38. One and one third times a number is -4.82 . What is the number?

GENETICS For Exercises 39–41, use the following information.

Research conducted by a daily U.S. newspaper has shown that about one seventh of people in the world are left-handed.

39. Write a multiplication equation relating the number of left-handed people ℓ and the total number of people p .
40. About how many left-handed people are there in a group of 350 people?
41. If there are 65 left-handed people in a group, about how many people are in that group?

42. **WORLD RECORDS** In 1993, a group of people in Utica, New York, made a very large round jelly doughnut which broke the world record for doughnut size. It weighed 1.5 tons and had a circumference of 50 feet. What was the diameter of the doughnut? (*Hint: $C = \pi d$*)

BASEBALL For Exercises 43–45, use the following information.

In baseball, if all other factors are the same, the speed of a four-seam fastball is faster than a two-seam fastball. The distance from the pitcher's mound to home plate is 60.5 feet.

43. How long does it take a two-seam fastball to go from the pitcher's mound to home plate? Round to the nearest hundredth. (*Hint: $rt = d$*)
44. How long does it take a four-seam fastball to go from the pitcher's mound to home plate? Round to the nearest hundredth.
45. How much longer does it take for a two-seam fastball to reach home plate than a four-seam fastball?

Two-Seam Fastball
126 ft/s



Four-Seam Fastball
132 ft/s



Source: Baseball and Mathematics

PHYSICAL SCIENCE For Exercises 46–49, use the following information.

In science lab, Devin and his classmates are asked to determine how many grams of hydrogen and how many grams of oxygen are in 477 grams of water. Devin used what he learned in class to determine that for every 8 grams of oxygen in water, there is 1 gram of hydrogen.

46. If x represents the number of grams of hydrogen, write an expression to represent the number of grams of oxygen.
47. Write an equation to represent the situation.
48. How many grams of hydrogen are in 477 grams of water?
49. How many grams of oxygen are in 477 grams of water?

50. **CRITICAL THINKING** If $6y - 7 = 4$, what is the value of $18y - 21$?



51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can equations be used to find how long it takes light to reach Earth?

Include the following in your answer:

- an explanation of how to find the length of time it takes light to reach Earth from the closest star in the Big Dipper, and
- an equation describing the situation for the furthest star in the Big Dipper.



52. The rectangle at the right is divided into 5 identical squares. If the perimeter of the rectangle is 48 inches, what is the area of each square?



- (A) 4 in^2 (B) 9.8 in^2 (C) 16 in^2 (D) 23.04 in^2
53. Which equation is equivalent to $4t = 20$?
- (A) $-2t = -10$ (B) $t = 80$ (C) $2t = 5$ (D) $-8t = 40$

Maintain Your Skills

Mixed Review Solve each equation. Then check your solution. (Lesson 3-2)

54. $m + 14 = 81$ 55. $d - 27 = -14$ 56. $17 - (-w) = -55$

57. Translate the following sentence into an equation. (Lesson 3-1)
Ten times a number a is equal to 5 times the sum of b and c .

Find each product. (Lesson 2-3)

58. $(-5)(12)$ 59. $(-2.93)(-0.003)$ 60. $(-4)(0)(-2)(-3)$

Graph each set of numbers on a number line. (Lesson 2-1)

61. $\{-4, -3, -1, 3\}$ 62. {integers between -6 and 10 }
 63. {integers less than -4 } 64. {integers less than 0 and greater than -6 }

Name the property illustrated by each statement. (Lesson 1-6)

65. $67 + 3 = 3 + 67$ 66. $(5 \cdot m) \cdot n = 5 \cdot (m \cdot n)$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Use the order of operations to find each value. (To review the order of operations, see Lesson 1-2.)

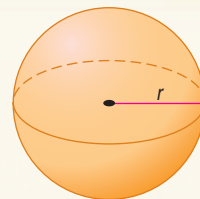
67. $2 \times 8 + 9$ 68. $24 \div 3 - 8$ 69. $\frac{3}{8}(17 + 7)$ 70. $\frac{15 - 9}{26 + 12}$

Practice Quiz 1

Lessons 3-1 through 3-3

GEOMETRY For Exercises 1 and 2, use the following information. The surface area S of a sphere equals four times π times the square of the radius r . (Lesson 3-1)

- Write the formula for the surface area of a sphere.
- What is the surface area of a sphere if the radius is 7 centimeters?



Solve each equation. Then check your solution. (Lessons 3-2 and 3-3)

3. $d + 18 = -27$ 4. $m - 77 = -61$ 5. $-12 + a = -36$ 6. $t - (-16) = 9$
 7. $\frac{2}{3}p = 18$ 8. $-17y = 391$ 9. $5x = -45$ 10. $-\frac{2}{5}d = -10$



Algebra Activity

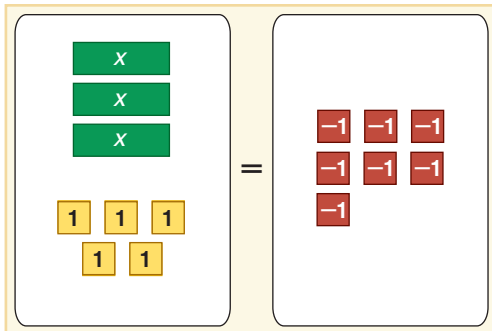
A Preview of Lesson 3-4

Solving Multi-Step Equations

You can use an equation model to solve multi-step equations.

Solve $3x + 5 = -7$.

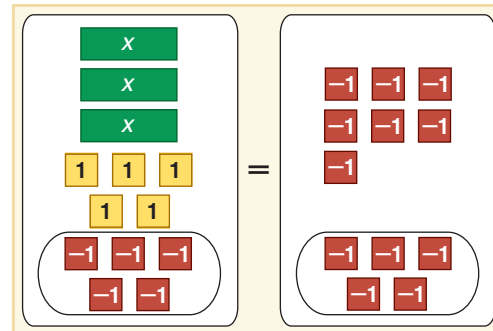
Step 1 Model the equation.



$$3x + 5 = -7$$

Place 3 x tiles and 5 positive 1 tiles on one side of the mat. Place 7 negative 1 tiles on the other side of the mat.

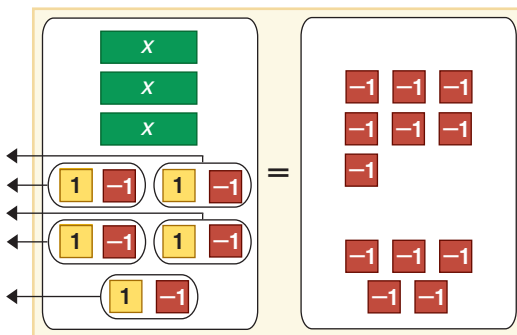
Step 2 Isolate the x term.



$$3x + 5 - 5 = -7 - 5$$

Since there are 5 positive 1 tiles with the x tiles, add 5 negative 1 tiles to each side to form zero pairs.

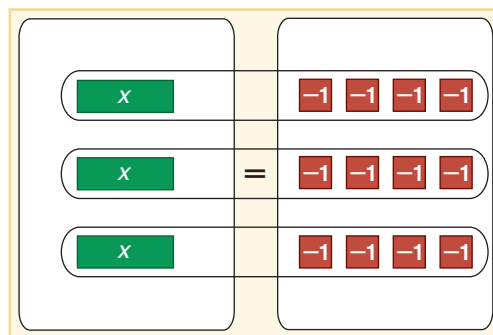
Step 3 Remove zero pairs.



$$3x = -12$$

Group the tiles to form zero pairs and remove the zero pairs.

Step 4 Group the tiles.



$$\frac{3x}{3} = \frac{-12}{3}$$

Separate the tiles into 3 equal groups to match the 3 x tiles. Each x tile is paired with 4 negative 1 tiles. Thus, $x = -4$.

Model Use algebra tiles to solve each equation.

1. $2x - 3 = -9$
2. $3x + 5 = 14$
3. $3x - 2 = 10$
4. $-8 = 2x + 4$
5. $3 + 4x = 11$
6. $2x + 7 = 1$
7. $9 = 4x - 7$
8. $7 + 3x = -8$

9. **MAKE A CONJECTURE** What steps would you use to solve $7x - 12 = -61$?

3-4

Solving Multi-Step Equations

What You'll Learn

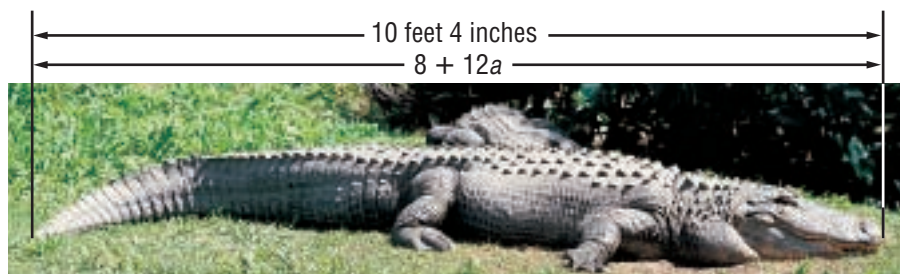
- Solve problems by working backward.
- Solve equations involving more than one operation.

Vocabulary

- work backward
- multi-step equations
- consecutive integers
- number theory

How can equations be used to estimate the age of an animal?

An American alligator hatchling is about 8 inches long. These alligators grow about 12 inches per year. Therefore, the expression $8 + 12a$ represents the length in inches of an alligator that is a years old.



Since 10 feet 4 inches equals $10(12) + 4$ or 124 inches, the equation $8 + 12a = 124$ can be used to estimate the age of the alligator in the photograph. Notice that this equation involves more than one operation.

WORK BACKWARD **Work backward** is one of many *problem-solving strategies* that you can use. Here are some other problem-solving strategies.

Problem-Solving Strategies	
draw a diagram	solve a simpler (or similar) problem
make a table or chart	eliminate the possibilities
make a model	look for a pattern
guess and check	act it out
check for hidden assumptions	list the possibilities
use a graph	identify the subgoals

Example 1 Work Backward to Solve a Problem

Solve the following problem by working backward.

After cashing her paycheck, Tara paid her father the \$20 she had borrowed. She then spent half of the remaining money on a concert ticket. She bought lunch for \$4.35 and had \$10.55 left. What was the amount of the paycheck?

Start at the end of the problem and undo each step.

Statement	Undo the Statement
She had \$10.55 left.	\$10.55
She bought lunch for \$4.35.	$\$10.55 + \$4.35 = \$14.90$
She spent half of the money on a concert ticket.	$\$14.90 \times 2 = \29.80
She paid her father \$20.	$\$29.80 + \$20.00 = \$49.80$

The paycheck was for \$49.80. *Check this answer in the context of the problem.*

SOLVE MULTI-STEP EQUATIONS To solve equations with more than one operation, often called **multi-step equations**, undo operations by working backward.

Study Tip

Solving Multi-Step Equations

When solving a multi-step equation, “undo” the operations in reverse of the order of operations.

Example 2 Solve Using Addition and Division

Solve $7m - 17 = 60$. Then check your solution.

$$7m - 17 = 60 \quad \text{Original equation}$$

$$7m - 17 + 17 = 60 + 17 \quad \text{Add 17 to each side.}$$

$$7m = 77 \quad \text{Simplify.}$$

$$\frac{7m}{7} = \frac{77}{7} \quad \text{Divide each side by 7.}$$

$$m = 11 \quad \text{Simplify.}$$

CHECK $7m - 17 = 60$ Original equation

$$7(11) - 17 \stackrel{?}{=} 60 \quad \text{Substitute 11 for } m.$$

$$77 - 17 \stackrel{?}{=} 60 \quad \text{Multiply.}$$

$$60 = 60 \checkmark \quad \text{The solution is 11.}$$

You have seen a multi-step equation in which the first, or *leading*, coefficient is an integer. You can use the same steps if the leading coefficient is a fraction.

Example 3 Solve Using Subtraction and Multiplication

Solve $\frac{t}{8} + 21 = 14$. Then check your solution.

$$\frac{t}{8} + 21 = 14 \quad \text{Original equation}$$

$$\frac{t}{8} + 21 - 21 = 14 - 21 \quad \text{Subtract 21 from each side.}$$

$$\frac{t}{8} = -7 \quad \text{Simplify.}$$

$$8\left(\frac{t}{8}\right) = 8(-7) \quad \text{Multiply each side by 8.}$$

$$t = -56 \quad \text{Simplify.}$$

CHECK $\frac{t}{8} + 21 = 14$ Original equation

$$\frac{-56}{8} + 21 \stackrel{?}{=} 14 \quad \text{Substitute } -56 \text{ for } t.$$

$$-7 + 21 \stackrel{?}{=} 14 \quad \text{Divide.}$$

$$14 = 14 \checkmark \quad \text{The solution is } -56.$$

Example 4 Solve Using Multiplication and Addition

Solve $\frac{p - 15}{9} = -6$.

$$\frac{p - 15}{9} = -6 \quad \text{Original equation}$$

$$9\left(\frac{p - 15}{9}\right) = 9(-6) \quad \text{Multiply each side by 9.}$$

$$p - 15 = -54 \quad \text{Simplify.}$$

$$p - 15 + 15 = -54 + 15 \quad \text{Add 15 to each side.}$$

$$p = -39 \quad \text{The solution is } -39.$$



Example 5 Write and Solve a Multi-Step Equation

Write an equation for the problem below. Then solve the equation.

Two-thirds of a number minus six is -10 .

$$\underbrace{\frac{2}{3}}_{\text{Two-thirds}} \cdot \underbrace{n}_{\text{of a number}} - \underbrace{6}_{\text{minus six}} = \underbrace{-10}_{\text{is } -10}$$

$$\frac{2}{3}n - 6 = -10 \quad \text{Original equation}$$

$$\frac{2}{3}n - 6 + 6 = -10 + 6 \quad \text{Add 6 to each side.}$$

$$\frac{2}{3}n = -4 \quad \text{Simplify.}$$

$$\frac{3}{2}\left(\frac{2}{3}n\right) = \frac{3}{2}(-4) \quad \text{Multiply each side by } \frac{3}{2}.$$

$$n = -6 \quad \text{Simplify.}$$

The solution is -6 .

Consecutive integers are integers in counting order, such as 7, 8, and 9.

Beginning with an even integer and counting by two will result in *consecutive even integers*. For example, $-4, -2, 0,$ and 2 are consecutive even integers. Beginning with an odd integer and counting by two will result in *consecutive odd integers*. For example, $-3, -1, 1, 3$ and 5 are consecutive odd integers. The study of numbers and the relationships between them is called **number theory**.

Example 6 Solve a Consecutive Integer Problem

NUMBER THEORY Write an equation for the problem below. Then solve the equation and answer the problem.

Find three consecutive even integers whose sum is -42 .

Let n = the least even integer.

Then $n + 2$ = the next greater even integer, and

$n + 4$ = the greatest of the three even integers.

$$\underbrace{n + (n + 2) + (n + 4)}_{\text{The sum of three consecutive even integers}} = \underbrace{-42}_{\text{is } -42}$$

$$n + (n + 2) + (n + 4) = -42 \quad \text{Original equation}$$

$$3n + 6 = -42 \quad \text{Simplify.}$$

$$3n + 6 - 6 = -42 - 6 \quad \text{Subtract 6 from each side.}$$

$$3n = -48 \quad \text{Simplify.}$$

$$\frac{3n}{3} = \frac{-48}{3} \quad \text{Divide each side by 3}$$

$$n = -16 \quad \text{Simplify.}$$

$$n + 2 = -16 + 2 \text{ or } -14 \quad n + 4 = -16 + 4 \text{ or } -12$$

The consecutive even integers are $-16, -14,$ and -12 .

CHECK $-16, -14,$ and -12 are consecutive even integers.
 $-16 + (-14) + (-12) = -42 \quad \checkmark$

Study Tip

Representing Consecutive Integers

You can use the same expressions to represent either consecutive even integers or consecutive odd integers. It is the value of n —odd or even—that differs between the two expressions.

Check for Understanding

- Concept Check**
- OPEN ENDED** Give two examples of multi-step equations that have a solution of -2 .
 - List the steps used to solve $\frac{w+3}{5} - 4 = 6$.
 - Write an expression for the odd integer before odd integer n .
 - Justify each step.

$\frac{4-2d}{5} + 3 = 9$	
$\frac{4-2d}{5} + 3 - 3 = 9 - 3$	a. <u> ?</u>
$\frac{4-2d}{5} = 6$	b. <u> ?</u>
$\frac{4-2d}{5}(5) = 6(5)$	c. <u> ?</u>
$4 - 2d = 30$	d. <u> ?</u>
$4 - 2d - 4 = 30 - 4$	e. <u> ?</u>
$-2d = 26$	f. <u> ?</u>
$\frac{-2d}{-2} = \frac{26}{-2}$	g. <u> ?</u>
$d = -13$	h. <u> ?</u>

Guided Practice Solve each problem by working backward.

- A number is multiplied by seven, and then the product is added to 13. The result is 55. What is the number?
- LIFE SCIENCE** A bacteria population triples in number each day. If there are 2,187,000 bacteria on the seventh day, how many bacteria were there on the first day?

Solve each equation. Then check your solution.

- | | |
|----------------------------|----------------------------|
| 7. $4g - 2 = -6$ | 8. $18 = 5p + 3$ |
| 9. $\frac{3}{2}a - 8 = 11$ | 10. $\frac{b+4}{-2} = -17$ |
| 11. $0.2n + 3 = 8.6$ | 12. $3.1y - 1.5 = 5.32$ |

Write an equation and solve each problem.

- Twelve decreased by a twice a number equals -34 . Find the number.
- Find three consecutive integers whose sum is 42.

- Application**
- WORLD CULTURES** The English alphabet contains 2 more than twice as many letters as the Hawaiian alphabet. How many letters are there in the Hawaiian alphabet?

Practice and Apply

Solve each problem by working backward.

- A number is divided by 4, and then the quotient is added to 17. The result is 25. Find the number.
- Nine is subtracted from a number, and then the difference is multiplied by 5. The result is 75. What is the number?



Homework Help

For Exercises	See Examples
16–21	1
22–41	2–4
42–54	5, 6

Extra Practice

See page 826.

Solve each problem by working backward.

- GAMES** In the Trivia Bowl, each finalist must answer four questions correctly. Each question is worth twice as much as the question before it. The fourth question is worth \$6000. How much is the first question worth?
- ICE SCULPTING** Due to melting, an ice sculpture loses one-half its weight every hour. After 8 hours, it weighs $\frac{5}{16}$ of a pound. How much did it weigh in the beginning?
- FIREFIGHTING** A firefighter spraying water on a fire stood on the middle rung of a ladder. The smoke lessened, so she moved up 3 rungs. It got too hot, so she backed down 5 rungs. Later, she went up 7 rungs and stayed until the fire was out. Then, she climbed the remaining 4 rungs and went into the building. How many rungs does the ladder have?
- MONEY** Hugo withdrew some money from his bank account. He spent one third of the money for gasoline. Then he spent half of what was left for a haircut. He bought lunch for \$6.55. When he got home, he had \$13.45 left. How much did he withdraw from the bank?

Solve each equation. Then check your solution.

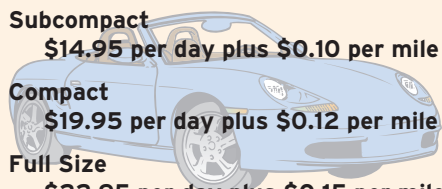
- $5n + 6 = -4$
 - $7 + 3c = -11$
 - $15 = 4a - 5$
 - $-63 = 7g - 14$
 - $\frac{c}{-3} + 5 = 7$
 - $\frac{y}{5} + 9 = 6$
 - $3 - \frac{a}{7} = -2$
 - $-9 - \frac{p}{4} = 5$
 - $\frac{t}{8} - 6 = -12$
 - $\frac{m}{-5} + 6 = 31$
 - $\frac{17 - s}{4} = -10$
 - $\frac{-3j - (-4)}{-6} = 12$
 - $-3d - 1.2 = 0.9$
 - $-2.5r - 32.7 = 74.1$
 - $-0.6 + (-4a) = -1.4$
 - $\frac{p}{-7} - 0.5 = 1.3$
 - $3.5x + 5 - 1.5x = 8$
 - $\frac{9z + 4}{5} - 8 = 5.4$
40. If $3a - 9 = 6$, what is the value of $5a + 2$?
41. If $2x + 1 = 5$, what is the value of $3x - 4$?

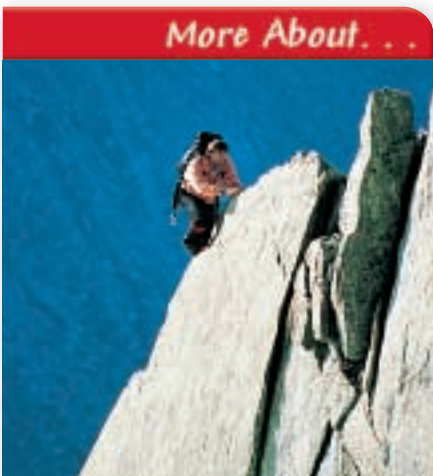
Write an equation and solve each problem.

- Six less than two thirds of a number is negative ten. Find the number.
- Twenty-nine is thirteen added to four times a number. What is the number?
- Find three consecutive odd integers whose sum is 51.
- Find three consecutive even integers whose sum is -30 .
- Find four consecutive integers whose sum is 94.
- Find four consecutive odd integers whose sum is 8.
- BUSINESS** Adele Jones is on a business trip and plans to rent a subcompact car from Speedy Rent-A-Car. Her company has given her a budget of \$60 per day for car rental. What is the maximum distance Ms. Jones can drive in one day and still stay within her budget?

Speedy Rent-A-Car Price List

Subcompact	\$14.95 per day plus \$0.10 per mile
Compact	\$19.95 per day plus \$0.12 per mile
Full Size	\$22.95 per day plus \$0.15 per mile





More About . . .

Mountain Climbing

Many mountain climbers experience altitude sickness caused by a decrease in oxygen. Climbers can acclimate themselves to these higher altitudes by camping for one or two weeks at various altitudes as they ascend the mountain.

Source: Shape

49. **GEOMETRY** The measures of the three sides of a triangle are consecutive even integers. The perimeter of the triangle is 54 centimeters. What are the lengths of the sides of the triangle?

50. **MOUNTAIN CLIMBING** A general rule for those climbing more than 7000 feet above sea level is to allow a total of $\left(\frac{a - 7000}{2000} + 2\right)$ weeks of camping during the ascension. In this expression, a represents the altitude in feet. If a group of mountain climbers have allowed for 9 weeks of camping in their schedule, how high can they climb without worrying about altitude sickness?

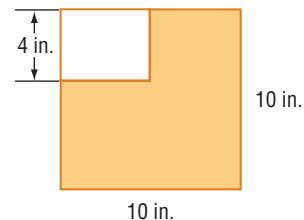
SHOE SIZE For Exercises 51 and 52, use the following information.

If ℓ represents the length of a person's foot in inches, the expression $2\ell - 12$ can be used to estimate his or her shoe size.

51. What is the approximate length of the foot of a person who wears size 8?
 52. Measure your foot and use the expression to determine your shoe size. How does this number compare to the size of shoe you are wearing?

53. **SALES** Trever Goetz is a salesperson who is paid a monthly salary of \$500 plus a 2% commission on sales. How much must Mr. Goetz sell to earn \$2000 this month?

54. **GEOMETRY** A rectangle is cut from the corner of a 10-inch by 10-inch of paper. The area of the remaining piece of paper is $\frac{4}{5}$ of the area of the original piece of paper. If the width of the rectangle removed from the paper is 4 inches, what is the length of the rectangle?



55. **CRITICAL THINKING** Determine whether the following statement is *sometimes*, *always*, or *never* true.

The sum of two consecutive even numbers equals the sum of two consecutive odd numbers.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can equations be used to estimate the age of an animal?

Include the following in your answer:

- an explanation of how to solve the equation representing the age of the alligator, and
- an estimate of the age of the alligator.

57. Which equation represents the following problem?

Fifteen minus three times a number equals negative twenty-two. Find the number.

- (A) $3n - 15 = -22$ (B) $15 - 3n = -22$
 (C) $3(15 - n) = -22$ (D) $3(n - 15) = -22$

58. Which equation has a solution of -5 ?

- (A) $2a - 6 = 4$ (B) $3a + 7 = 8$
 (C) $\frac{3a - 7}{4} = 2$ (D) $\frac{3}{5}a + 19 = 16$





Graphing Calculator

EQUATION SOLVER You can use a graphing calculator to solve equations that are rewritten as expressions that equal zero.

Step 1 Write the equation so that one side is equal to 0.

Step 2 On a TI-83 Plus, press **MATH** and choose 0, for solve.

Step 3 Enter the equation after 0=. Use **ALPHA** to enter the variables. Press **ENTER**.

Step 4 Press **ALPHA** [SOLVE] to reveal the solution. Use the **▲** key to begin entering a new equation.

Use a graphing calculator to solve each equation.

59. $0 = 11y + 33$

60. $\frac{w+2}{5} - 4 = 0$

61. $6 = -12 + \frac{h}{-7}$

62. $\frac{p - (-5)}{-2} = 6$

63. $0.7 = \frac{r - 0.8}{6}$

64. $4.91 + 7.2t = 38.75$

Maintain Your Skills

Mixed Review Solve each equation. Then check your solution. (Lesson 3-3)

65. $-7t = 91$

66. $\frac{r}{15} = -8$

67. $-\frac{2}{3}b = -1\frac{1}{2}$

TRANSPORTATION For Exercises 68 and 69, use the following information.

In the year 2000, there were 18 more models of sport utility vehicles than there were in the year 1990. There were 47 models of sport utility vehicles in 2000. (Lesson 3-2)

68. Write an addition equation to represent the situation.

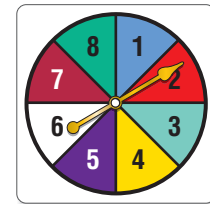
69. How many models of sport utility vehicles were there in 1990?

Find the odds of each outcome if you spin the spinner at the right. (Lesson 2-6)

70. spinning a number divisible by 3

71. spinning a number equal to or greater than 5

72. spinning a number less than 7



Find each quotient. (Lesson 2-4)

73. $-\frac{6}{7} \div 3$

74. $\frac{2}{3} \div \frac{8}{8}$

75. $\frac{-3a + 16}{4}$

76. $\frac{15t - 25}{-5}$

Use the Distributive Property to find each product. (Lesson 1-5)

77. $17 \cdot 9$

78. $13(101)$

79. $16\left(1\frac{1}{4}\right)$

80. $18\left(2\frac{1}{9}\right)$

Write an algebraic expression for each verbal expression. (Lesson 1-1)

81. the product of 5 and m plus half of n

82. the sum of 3 and b divided by y

83. the sum of 3 times a and the square of b

Getting Ready for the Next Lesson

PREREQUISITE SKILL Simplify each expression.

(To review simplifying expressions, see Lesson 1-5.)

84. $5d - 2d$

85. $11m - 5m$

86. $8t + 6t$

87. $7g - 15g$

88. $-9f + 6f$

89. $-3m + (-7m)$



Solving Equations with the Variable on Each Side

What You'll Learn

- Solve equations with the variable on each side.
- Solve equations involving grouping symbols.

Vocabulary

- identity

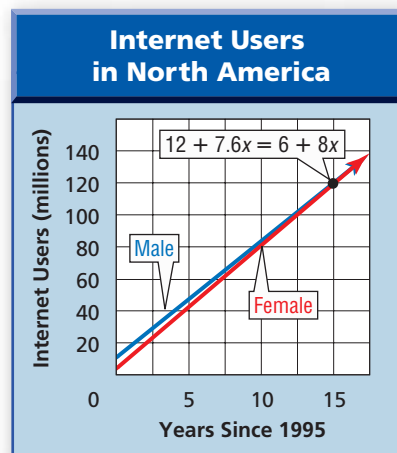
How can an equation be used to determine when two populations are equal?

In 1995, there were 18 million Internet users in North America. Of this total, 12 million were male, and 6 million were female. During the next five years, the number of male Internet users on average increased 7.6 million per year, and the number of female Internet users increased 8 million per year. If this trend continues, the following expressions represent the number of male and female Internet users x years after 1995.

$$\text{Male Internet Users: } 12 + 7.6x$$

$$\text{Female Internet Users: } 6 + 8x$$

The equation $12 + 7.6x = 6 + 8x$ represents the time at which the number of male and female Internet users are equal. Notice that this equation has the variable x on each side.



VARIABLES ON EACH SIDE Many equations contain variables on each side. To solve these types of equations, first use the Addition or Subtraction Property of Equality to write an equivalent equation that has all of the variables on one side.

Example 1 Solve an Equation with Variables on Each Side

Solve $-2 + 10p = 8p - 1$. Then check your solution.

$$-2 + 10p = 8p - 1 \quad \text{Original equation}$$

$$-2 + 10p - 8p = 8p - 1 - 8p \quad \text{Subtract } 8p \text{ from each side.}$$

$$-2 + 2p = -1 \quad \text{Simplify.}$$

$$-2 + 2p + 2 = -1 + 2 \quad \text{Add 2 to each side.}$$

$$2p = 1 \quad \text{Simplify.}$$

$$\frac{2p}{2} = \frac{1}{2} \quad \text{Divide each side by 2.}$$

$$p = \frac{1}{2} \text{ or } 0.5 \quad \text{Simplify.}$$

CHECK $-2 + 10p = 8p - 1$ Original equation

$$-2 + 10(0.5) \stackrel{?}{=} 8(0.5) - 1 \quad \text{Substitute 0.5 for } p.$$

$$-2 + 5 \stackrel{?}{=} 4 - 1 \quad \text{Multiply.}$$

$$3 = 3 \quad \checkmark \quad \text{The solution is } \frac{1}{2} \text{ or } 0.5.$$

GROUPING SYMBOLS When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example 2 Solve an Equation with Grouping Symbols

Solve $4(2r - 8) = \frac{1}{7}(49r + 70)$. Then check your solution.

$$4(2r - 8) = \frac{1}{7}(49r + 70) \quad \text{Original equation}$$

$$8r - 32 = 7r + 10 \quad \text{Distributive Property}$$

$$8r - 32 - 7r = 7r + 10 - 7r \quad \text{Subtract } 7r \text{ from each side.}$$

$$r - 32 = 10 \quad \text{Simplify.}$$

$$r - 32 + 32 = 10 + 32 \quad \text{Add 32 to each side.}$$

$$r = 42 \quad \text{Simplify.}$$

CHECK $4(2r - 8) = \frac{1}{7}(49r + 70)$ Original equation

$$4[2(42) - 8] \stackrel{?}{=} \frac{1}{7}[49(42) + 70] \quad \text{Substitute 42 for } r.$$

$$4(84 - 8) \stackrel{?}{=} \frac{1}{7}(2058 + 70) \quad \text{Multiply.}$$

$$4(76) \stackrel{?}{=} \frac{1}{7}(2128) \quad \text{Add and subtract.}$$

$$304 = 304 \quad \checkmark$$

The solution is 42.

Some equations with the variable on each side may have no solution. That is, there is no value of the variable that will result in a true equation.

Example 3 No Solutions

Solve $2m + 5 = 5(m - 7) - 3m$.

$$2m + 5 = 5(m - 7) - 3m \quad \text{Original equation}$$

$$2m + 5 = 5m - 35 - 3m \quad \text{Distributive Property}$$

$$2m + 5 = 2m - 35 \quad \text{Simplify.}$$

$$2m + 5 - 2m = 2m - 35 - 2m \quad \text{Subtract } 2m \text{ from each side.}$$

$$5 = -35 \quad \text{This statement is false.}$$

Since $5 = -35$ is a false statement, this equation has no solution.

An equation that is true for every value of the variable is called an **identity**.

Example 4 An Identity

Solve $3(r + 1) - 5 = 3r - 2$.

$$3(r + 1) - 5 = 3r - 2 \quad \text{Original equation}$$

$$3r + 3 - 5 = 3r - 2 \quad \text{Distributive Property}$$

$$3r - 2 = 3r - 2 \quad \text{Reflexive Property of Equality}$$

Since the expressions on each side of the equation are the same, this equation is an identity. The statement $3(r + 1) - 5 = 3r - 2$ is true for all values of r .

Study Tip

Look Back

To review the **Distributive Property**, see Lesson 1-5.

- Step 1** Use the Distributive Property to remove the grouping symbols.
- Step 2** Simplify the expressions on each side of the equals sign.
- Step 3** Use the Addition and/or Subtraction Properties of Equality to get the variables on one side of the equals sign and the numbers without variables on the other side of the equals sign.
- Step 4** Simplify the expressions on each side of the equals sign.
- Step 5** Use the Multiplication or Division Property of Equality to solve.
 - If the solution results in a false statement, there is no solution of the equation.
 - If the solution results in an identity, the solution is all numbers.

Standardized Test Practice

A B C D

Example 5 Use Substitution to Solve an Equation

Multiple-Choice Test Item

Solve $2(b - 3) + 5 = 3(b - 1)$.

- (A) -2 (B) 2 (C) -3 (D) 3

Read the Test Item

You are asked to solve an equation.

Solve the Test Item

You can solve the equation or substitute each value into the equation and see if it makes the equation true. We will solve by substitution.

- | | |
|---|--|
| <p>A $2(b - 3) + 5 = 3(b - 1)$</p> <p>$2(-2 - 3) + 5 \stackrel{?}{=} 3(-2 - 1)$</p> <p>$2(-5) + 5 \stackrel{?}{=} 3(-3)$</p> <p>$-10 + 5 \stackrel{?}{=} -9$</p> <p>$-5 \neq -9$</p> | <p>B $2(b - 3) + 5 = 3(b - 1)$</p> <p>$2(2 - 3) + 5 \stackrel{?}{=} 3(2 - 1)$</p> <p>$2(-1) + 5 \stackrel{?}{=} 3(1)$</p> <p>$-2 + 5 \stackrel{?}{=} 3$</p> <p>$3 = 3 \quad \checkmark$</p> |
|---|--|

Since the value 2 results in a true statement, you do not need to check -3 and 3. The answer is B.



Test-Taking Tip

If you are asked to solve a complicated equation, it sometimes takes less time to check each possible answer rather than to actually solve the equation.

Check for Understanding

Concept Check

1. Determine whether each solution is correct. If the solution is not correct, find the error and give the correct solution.
- | | | |
|--|---|---|
| <p>a. $2(g + 5) = 22$</p> <p>$2g + 5 = 22$</p> <p>$2g + 5 - 5 = 22 - 5$</p> <p>$2g = 17$</p> <p>$\frac{2g}{2} = \frac{17}{2}$</p> <p>$g = 8.5$</p> | <p>b. $5d = 2d - 18$</p> <p>$5d - 2d = 2d - 18 - 2d$</p> <p>$3d = -18$</p> <p>$\frac{3d}{3} = \frac{-18}{3}$</p> <p>$d = -6$</p> | <p>c. $-6z + 13 = 7z$</p> <p>$-6z + 13 - 6z = 7z - 6z$</p> <p>$13 = z$</p> |
|--|---|---|



2. Explain how to determine whether an equation is an identity.
3. **OPEN ENDED** Find a counterexample to the statement *all equations have a solution*.

Guided Practice

4. Justify each step.
- $$6n + 7 = 8n - 13$$
- $$6n + 7 - 6n = 8n - 13 - 6n \quad \text{a. } \underline{\quad ? \quad}$$
- $$7 = 2n - 13 \quad \text{b. } \underline{\quad ? \quad}$$
- $$7 + 13 = 2n - 13 + 13 \quad \text{c. } \underline{\quad ? \quad}$$
- $$20 = 2n \quad \text{d. } \underline{\quad ? \quad}$$
- $$\frac{20}{2} = \frac{2n}{2} \quad \text{e. } \underline{\quad ? \quad}$$
- $$10 = n \quad \text{f. } \underline{\quad ? \quad}$$

Solve each equation. Then check your solution.

5. $20c + 5 = 5c + 65$
6. $\frac{3}{8} - \frac{1}{4}t = \frac{1}{2}t - \frac{3}{4}$
7. $3(a - 5) = -6$
8. $7 - 3r = r - 4(2 + r)$
9. $6 = 3 + 5(d - 2)$
10. $\frac{c + 1}{8} = \frac{c}{4}$
11. $5h - 7 = 5(h - 2) + 3$
12. $5.4w + 8.2 = 9.8w - 2.8$
13. Solve $75 - 9t = 5(-4 + 2t)$.
- (A) -5 (B) -4 (C) 4 (D) 5

Standardized Test Practice

Practice and Apply

Homework Help

For Exercises	See Examples
14–48	1–4
51, 52	5

Extra Practice

See page 826.

Justify each step.

14. $\frac{3m - 2}{5} = \frac{7}{10}$
- $$\frac{3m - 2}{5} (10) = \frac{7}{10} (10) \quad \text{a. } \underline{\quad ? \quad}$$
- $$(3m - 2)2 = 7 \quad \text{b. } \underline{\quad ? \quad}$$
- $$6m - 4 = 7 \quad \text{c. } \underline{\quad ? \quad}$$
- $$6m - 4 + 4 = 7 + 4 \quad \text{d. } \underline{\quad ? \quad}$$
- $$6m = 11 \quad \text{e. } \underline{\quad ? \quad}$$
- $$\frac{6m}{6} = \frac{11}{6} \quad \text{f. } \underline{\quad ? \quad}$$
- $$m = 1\frac{5}{6} \quad \text{g. } \underline{\quad ? \quad}$$
15. $v + 9 = 7v + 9$
- $$v + 9 - v = 7v + 9 - v \quad \text{a. } \underline{\quad ? \quad}$$
- $$9 = 6v + 9 \quad \text{b. } \underline{\quad ? \quad}$$
- $$9 - 9 = 6v + 9 - 9 \quad \text{c. } \underline{\quad ? \quad}$$
- $$0 = 6v \quad \text{d. } \underline{\quad ? \quad}$$
- $$\frac{0}{6} = \frac{6v}{6} \quad \text{e. } \underline{\quad ? \quad}$$
- $$0 = v \quad \text{f. } \underline{\quad ? \quad}$$

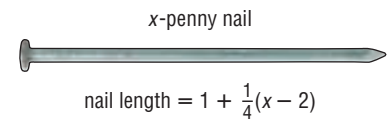
Solve each equation. Then check your solution.

16. $3 - 4q = 10q + 10$
17. $3k - 5 = 7k - 21$
18. $5t - 9 = -3t + 7$
19. $8s + 9 = 7s + 6$
20. $\frac{3}{4}n + 16 = 2 - \frac{1}{8}n$
21. $\frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y$
22. $8 = 4(3c + 5)$
23. $7(m - 3) = 7$
24. $6(r + 2) - 4 = -10$
25. $5 - \frac{1}{2}(x - 6) = 4$
26. $4(2a - 1) = -10(a - 5)$
27. $4(f - 2) = 4f$
28. $3(1 + d) - 5 = 3d - 2$
29. $2(w - 3) + 5 = 3(w - 1)$

30. $\frac{3}{2}y - y = 4 + \frac{1}{2}y$
31. $3 + \frac{2}{5}b = 11 - \frac{2}{5}b$
32. $\frac{1}{4}(7 + 3g) = -\frac{g}{8}$
33. $\frac{1}{6}(a - 4) = \frac{1}{3}(2a + 4)$
34. $28 - 2.2x = 11.6x + 262.6$
35. $1.03p - 4 = -2.15p + 8.72$
36. $18 - 3.8t = 7.36 - 1.9t$
37. $13.7v - 6.5 = -2.3v + 8.3$
38. $2[s + 3(s - 1)] = 18$
39. $-3(2n - 5) = 0.5(-12n + 30)$

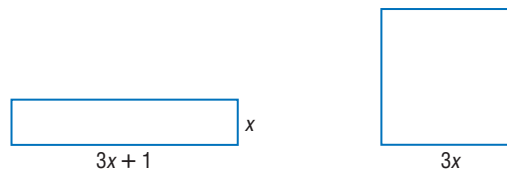
40. One half of a number increased by 16 is four less than two thirds of the number. Find the number.
41. The sum of one half of a number and 6 equals one third of the number. What is the number?
42. **NUMBER THEORY** Twice the greater of two consecutive odd integers is 13 less than three times the lesser number. Find the integers.
43. **NUMBER THEORY** Three times the greatest of three consecutive even integers exceeds twice the least by 38. What are the integers?
44. **HEALTH** When exercising, a person's pulse rate should not exceed a certain limit, which depends on his or her age. This maximum rate is represented by the expression $0.8(220 - a)$, where a is age in years. Find the age of a person whose maximum pulse is 152.

45. **HARDWARE** Traditionally, nails are given names such as 2-penny, 3-penny, and so on. These names describe the lengths of the nails. What is the name of a nail that is $2\frac{1}{2}$ inches long?

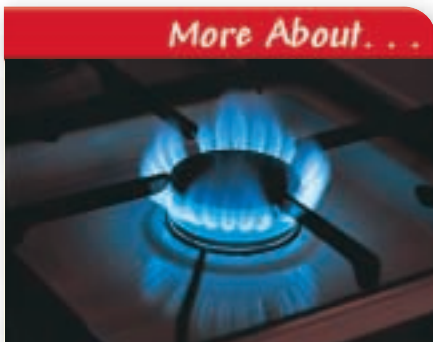


Source: *World Book Encyclopedia*

46. **TECHNOLOGY** About 4.9 million households had one brand of personal computers in 2001. The use of these computers grew at an average rate of 0.275 million households a year. In 2001, about 2.5 million households used another type of computer. The use of these computers grew at an average rate of 0.7 million households a year. How long will it take for the two types of computers to be in the same number of households?
47. **GEOMETRY** The rectangle and square shown below have the same perimeter. Find the dimensions of each figure.



48. **ENERGY** Use the information on energy at the left. The amount of energy E in BTUs needed to raise the temperature of water is represented by the equation $E = w(t_f - t_o)$. In this equation, w represents the weight of the water in pounds, t_f represents the final temperature in degrees Fahrenheit, and t_o represents the original temperature in degrees Fahrenheit. A 50-gallon water heater is 60% efficient. If 10 cubic feet of natural gas are used to raise the temperature of water with the original temperature of 50°F , what is the final temperature of the water? (One gallon of water weighs about 8 pounds.)
49. **CRITICAL THINKING** Write an equation that has one or more grouping symbols, the variable on each side of the equals sign, and a solution of -2 .



More About . . .

Energy

One British Thermal Unit (BTU) is the amount of energy needed to raise the temperature of one pound of water 1°F . If a heating system is 100% efficient, one cubic foot of natural gas provides 1000 BTU.

Source: *World Book Encyclopedia*



50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can an equation be used to determine when two populations are equal?

Include the following in your answer:

- a list of the steps needed to solve the equation,
- the year when the number of female Internet users will equal the number of male Internet users according to the model, and
- an explanation of why this method can be used to predict future events.



51. Solve $8x - 3 = 5(2x + 1)$.
(A) 4 (B) 2 (C) -2 (D) -4
52. Solve $5n + 4 = 7(n + 1) - 2n$.
(A) 0 (B) -1 (C) no solution (D) all numbers

Maintain Your Skills

Mixed Review

Solve each equation. Then check your solution. (Lesson 3-4)

53. $\frac{2}{9}v - 6 = 14$ 54. $\frac{x-3}{7} = -2$ 55. $5 - 9w = 23$

HEALTH For Exercises 56 and 57, use the following information.

Ebony burns 4.5 Calories per minute pushing a lawn mower. (Lesson 3-3)

56. Write a multiplication equation representing the number of Calories C burned if Ebony pushes the lawn mower for m minutes.
57. How long will it take Ebony to burn 150 Calories mowing the lawn?

Use each set of data to make a line plot. (Lesson 2-5)

58. 13, 15, 11, 15, 16, 17, 12, 12, 13, 15, 16, 15
59. 22, 25, 19, 21, 22, 24, 22, 25, 28, 21, 24, 22

Find each sum or difference. (Lesson 2-2)

60. $-10 + (-17)$ 61. $-12 - (-8)$ 62. $6 - 14$

Write a counterexample for each statement. (Lesson 1-7)

63. If the sum of two numbers is even, then both addends are even.
64. If you are baking cookies, you will need chocolate chips.

Evaluate each expression when $a = 5$, $b = 8$, $c = 7$, $x = 2$, and $y = 1$. (Lesson 1-2)

65. $\frac{3a^2}{b+c}$ 66. $x(a+2b) - y$ 67. $5(x+2y) - 4a$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Simplify each fraction.

(To review *simplifying fractions*, see pages 798 and 799.)

68. $\frac{12}{15}$ 69. $\frac{28}{49}$ 70. $\frac{36}{60}$ 71. $\frac{8}{120}$
72. $\frac{108}{9}$ 73. $\frac{28}{42}$ 74. $\frac{16}{40}$ 75. $\frac{19}{57}$



3-6 Ratios and Proportions

What You'll Learn

- Determine whether two ratios form a proportion.
- Solve proportions.

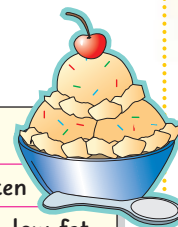
Vocabulary

- ratio
- proportion
- extremes
- means
- rate
- scale

How are ratios used in recipes?

The ingredients in the recipe will make 4 servings of honey frozen yogurt. Keri can use ratios and equations to find the amount of each ingredient needed to make enough yogurt for her club meeting.

Honey Frozen Yogurt	
2 cups 2% milk	2 eggs, beaten
$\frac{3}{4}$ cup honey	2 cups plain low-fat yogurt
1 dash salt	1 tablespoon vanilla



RATIOS AND PROPORTIONS A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed in the following ways.

$$x \text{ to } y \qquad x:y \qquad \frac{x}{y}$$

Ratios are often expressed in simplest form. For example, the recipe above states that for 4 servings you need 2 cups of milk. The ratio of servings to milk may be written as 4 to 2, 4:2, or $\frac{4}{2}$. Written in simplest form, the ratio of servings to milk can be written as 2 to 1, 2:1, or $\frac{2}{1}$.

Suppose you wanted to double the recipe to have 8 servings. The amount of milk required would be 4 cups. The ratio of servings to milk is $\frac{8}{4}$. When this ratio is simplified, the ratio is $\frac{2}{1}$. Notice that this ratio is equal to the original ratio.

$$\left[\begin{array}{c} \div 2 \\ \frac{4}{2} = \frac{2}{1} \\ \div 2 \end{array} \right]$$

$$\left[\begin{array}{c} \div 4 \\ \frac{8}{4} = \frac{2}{1} \\ \div 4 \end{array} \right]$$

An equation stating that two ratios are equal is called a **proportion**. So, we can state that $\frac{4}{2} = \frac{8}{4}$ is a proportion.

Example 1 Determine Whether Ratios Form a Proportion

Determine whether the ratios $\frac{4}{5}$ and $\frac{24}{30}$ form a proportion.

$$\left[\begin{array}{c} \div 1 \\ \frac{4}{5} = \frac{4}{5} \\ \div 1 \end{array} \right]$$

$$\left[\begin{array}{c} \div 6 \\ \frac{24}{30} = \frac{4}{5} \\ \div 6 \end{array} \right]$$

The ratios are equal. Therefore, they form a proportion.

Study Tip

Reading Math

A ratio that is equivalent to a whole number is written with a denominator of 1.

Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

Example 2 Use Cross Products

Use cross products to determine whether each pair of ratios form a proportion.

a. $\frac{0.4}{0.8} = \frac{0.7}{1.4}$

$$\frac{0.4}{0.8} \stackrel{?}{=} \frac{0.7}{1.4} \quad \text{Write the equation.}$$

$$0.4(1.4) \stackrel{?}{=} 0.8(0.7) \quad \text{Find the cross products.}$$

$$0.56 = 0.56 \quad \text{Simplify.}$$

The cross products are equal, so $\frac{0.4}{0.8} = \frac{0.7}{1.4}$. Since the ratios are equal, they form a proportion.

b. $\frac{6}{8} = \frac{24}{28}$

$$\frac{6}{8} \stackrel{?}{=} \frac{24}{28} \quad \text{Write the equation.}$$

$$6(28) \stackrel{?}{=} 8(24) \quad \text{Find the cross products.}$$

$$168 \neq 192 \quad \text{Simplify.}$$

The cross products are not equal, so $\frac{6}{8} \neq \frac{24}{28}$. The ratios do not form a proportion.

In the proportion $\frac{0.4}{0.8} = \frac{0.7}{1.4}$ above, 0.4 and 1.4 are called the **extremes**, and 0.8 and 0.7 are called the **means**.

Key Concept

Means-Extremes Property of Proportion

- **Words** In a proportion, the product of the extremes is equal to the product of the means.
- **Symbols** If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.
- **Examples** Since $\frac{2}{4} = \frac{1}{2}$, $2(2) = 4(1)$ or $4 = 4$.

SOLVE PROPORTIONS You can write proportions that involve a variable. To solve the proportion, use cross products and the techniques used to solve other equations.

Example 3 Solve a Proportion

Solve the proportion $\frac{n}{15} = \frac{24}{16}$.

$$\frac{n}{15} = \frac{24}{16} \quad \text{Original equation}$$

$$16(n) = 15(24) \quad \text{Find the cross products.}$$

$$16n = 360 \quad \text{Simplify.}$$

$$\frac{16n}{16} = \frac{360}{16} \quad \text{Divide each side by 16.}$$

$$n = 22.5 \quad \text{Simplify.}$$

Study Tip

Cross Products

When you find cross products, you are said to be *cross multiplying*.

The ratio of two measurements having different units of measure is called a **rate**. For example, a price of \$1.99 per dozen eggs, a speed of 55 miles per hour, and a salary of \$30,000 per year are all rates. Proportions are often used to solve problems involving rates.

Example 4 Use Rates

BICYCLING Trent goes on a 30-mile bike ride every Saturday. He rides the distance in 4 hours. At this rate, how far can he ride in 6 hours?

Explore Let m represent the number of miles Trent can ride in 6 hours.

Plan Write a proportion for the problem.

$$\begin{array}{l} \text{miles} \rightarrow \frac{30}{4} = \frac{m}{6} \leftarrow \text{miles} \\ \text{hours} \rightarrow \frac{4}{6} = \frac{6}{m} \leftarrow \text{hours} \end{array}$$

Solve $\frac{30}{4} = \frac{m}{6}$ Original proportion

$$30(6) = 4(m) \quad \text{Find the cross products.}$$

$$180 = 4m \quad \text{Simplify.}$$

$$\frac{180}{4} = \frac{4m}{4} \quad \text{Divide each side by 4.}$$

$$45 = m \quad \text{Simplify.}$$

Examine If Trent rides 30 miles in 4 hours, he rides 7.5 miles in 1 hour. So, in 6 hours, Trent can ride 6×7.5 or 45 miles. The answer is correct.

Since the rates are equal, they form a proportion. So, Trent can ride 45 miles in 6 hours.

A ratio or rate called a **scale** is used when making a model or drawing of something that is too large or too small to be conveniently drawn at actual size. The scale compares the model to the actual size of the object using a proportion. Maps and blueprints are two commonly used scale drawings.

Example 5 Use a Scale Drawing

CRATER LAKE The scale of a map for Crater Lake National Park is 2 inches = 9 miles. The distance between Discovery Point and Phantom Ship Overlook on the map is about $1\frac{3}{4}$ inches. What is the distance between these two places?

Let d represent the actual distance.

$$\begin{array}{l} \text{scale} \rightarrow \frac{2}{9} = \frac{1\frac{3}{4}}{d} \leftarrow \text{scale} \\ \text{actual} \rightarrow \frac{1\frac{3}{4}}{d} = \frac{2}{9} \leftarrow \text{actual} \end{array}$$

$$2(d) = 9\left(1\frac{3}{4}\right) \quad \text{Find the cross products.}$$

$$2d = \frac{63}{4} \quad \text{Simplify.}$$

$$2d \div 2 = \frac{63}{4} \div 2 \quad \text{Divide each side by 2.}$$

$$d = \frac{63}{8} \text{ or } 7\frac{7}{8} \quad \text{Simplify.}$$

The actual distance is about $7\frac{7}{8}$ miles.



Crater Lake

Crater Lake is a volcanic crater in Oregon that was formed by an explosion 42 times the blast of Mount St. Helens.

Source: travel.excite.com



Check for Understanding

- Concept Check**
- OPEN ENDED** Find an example of ratios used in advertisements.
 - Explain the difference between a ratio and a proportion.
 - Describe how to solve a proportion if one of the ratios contains a variable.

Guided Practice Use cross products to determine whether each pair of ratios form a proportion. Write *yes* or *no*.

4. $\frac{4}{11'} = \frac{12}{33}$

5. $\frac{16}{17'} = \frac{8}{9}$

6. $\frac{2.1}{3.5'} = \frac{0.5}{0.7}$

Solve each proportion. If necessary, round to the nearest hundredth.

7. $\frac{3}{4} = \frac{6}{x}$

8. $\frac{a}{45} = \frac{5}{15}$

9. $\frac{0.6}{1.1} = \frac{n}{8.47}$

- Application**
- TRAVEL** The Lehmans' minivan requires 5 gallons of gasoline to travel 120 miles. How much gasoline will they need for a 350-mile trip?

Practice and Apply

Homework Help

For Exercises	See Examples
11–18	1, 2
19–30	3
31, 32	4
33, 34	5

Use cross products to determine whether each pair of ratios form a proportion. Write *yes* or *no*.

11. $\frac{3}{2'} = \frac{21}{14}$

12. $\frac{8}{9'} = \frac{12}{18}$

13. $\frac{2.3}{3.4'} = \frac{3.0}{3.6}$

14. $\frac{4.2}{5.6'} = \frac{1.68}{2.24}$

15. $\frac{21.1}{14.4'} = \frac{1.1}{1.2}$


16. $\frac{5}{2'} = \frac{4}{1.6}$

Extra Practice

See page 827.

SPORTS For Exercises 17 and 18, use the graph at the right.

- Write a ratio of the number of gold medals won to the total number of medals won for each country.
- Do any two of the ratios you wrote for Exercise 17 form a proportion? If so, explain the real-world meaning of the proportion.



USA TODAY Snapshots®

USA stands atop all-time medals table

The USA, which led the 2000 Summer Olympics with 97 medals, has dominated the medal standings over the years. The all-time Summer Olympics medal standings:

	Gold	Silver	Bronze	Total
USA	871	659	586	2,116
USSR/Russia ¹	498	409	371	1,278
Germany ²	374	392	416	1,182
Great Britain	180	233	225	638
France	188	193	217	598
Italy	179	143	157	479
Sweden	136	156	177	469

1 – Competed as the Unified Team in 1992 after the breakup of the Soviet Union
2 – Totals include medals won by both East and West Germany.
Source: The Ultimate Book of Sports Lists

By Ellen J. Horrow and Marcy E. Mullins, USA TODAY

Solve each proportion. If necessary, round to the nearest hundredth.

19. $\frac{4}{x} = \frac{2}{10}$

20. $\frac{1}{y} = \frac{3}{15}$

21. $\frac{6}{5} = \frac{x}{15}$

22. $\frac{20}{28} = \frac{n}{21}$

23. $\frac{6}{8} = \frac{7}{a}$

24. $\frac{16}{7} = \frac{9}{b}$

25. $\frac{1}{0.19} = \frac{12}{n}$

26. $\frac{2}{0.21} = \frac{8}{n}$

27. $\frac{2.405}{3.67} = \frac{s}{1.88}$

28. $\frac{7}{1.066} = \frac{z}{9.65}$

29. $\frac{6}{14} = \frac{7}{x-3}$

30. $\frac{5}{3} = \frac{6}{x+2}$

WebQuest

A percent of increase or decrease can be used to describe trends in populations. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

31. **WORK** Seth earns \$152 in 4 days. At that rate, how many days will it take him to earn \$532?
32. **DRIVING** Lanette drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles?
33. **BLUEPRINTS** A blueprint for a house states that 2.5 inches equals 10 feet. If the length of a wall is 12 feet, how long is the wall in the blueprint?
34. **MODELS** A collector's model racecar is scaled so that 1 inch on the model equals $6\frac{1}{4}$ feet on the actual car. If the model is $\frac{2}{3}$ inch high, how high is the actual car?
35. **PETS** A research study shows that three out of every twenty pet owners got their pet from a breeder. Of the 122 animals cared for by a veterinarian, how many would you expect to have been bought from a breeder?
36. **CRITICAL THINKING** Consider the proportion $a:b:c = 3:1:5$. What is the value of $\frac{2a + 3b}{4b + 3c}$? (Hint: Choose different values of a , b , and c for which the proportion is true and evaluate the expression.)
37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are ratios used in recipes?

Include the following in your answer:

- an explanation of how to use a proportion to determine how much honey is needed if you use 3 eggs, and
- a description of how to alter the recipe to get 5 servings.

Standardized Test Practice

A B C D

38. Which ratio is *not* equal to $\frac{9}{12}$?

(A) $\frac{18}{24}$

(B) $\frac{3}{4}$

(C) $\frac{15}{20}$

(D) $\frac{18}{27}$

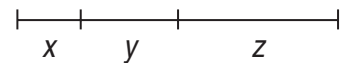
39. In the figure at the right, $x:y = 2:3$ and $y:z = 3:5$. If $x = 10$, find the value of z .

(A) 15

(B) 20

(C) 25

(D) 30



Maintain Your Skills

Mixed Review

Solve each equation. Then check your solution. (Lessons 3-4 and 3-5)

40. $8y - 10 = -3y + 2$

41. $17 + 2n = 21 + 2n$

42. $-7(d - 3) = -4$

43. $5 - 9w = 23$

44. $\frac{m}{-5} + 6 = 31$

45. $\frac{z - 7}{5} = -3$

Find each product. (Lesson 2-3)

46. $(-7)(-6)$

47. $(-\frac{8}{9})(\frac{9}{8})$

48. $(\frac{3}{7})(\frac{3}{7})$

49. $(-0.075)(-5.5)$

Find each absolute value. (Lesson 2-1)

50. $|-33|$

51. $|77|$

52. $|2.5|$

53. $|-0.85|$

54. Sketch a reasonable graph for the temperature in the following statement.
In August, you enter a hot house and turn on the air conditioner. (Lesson 1-9)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each percent. (To review percents, see pages 802 and 803.)

55. Eighteen is what percent of 60?

56. What percent of 14 is 4.34?

57. Six is what percent of 15?

58. What percent of 2 is 8?



3-7 Percent of Change

What You'll Learn

- Find percents of increase and decrease.
- Solve problems involving percents of change.

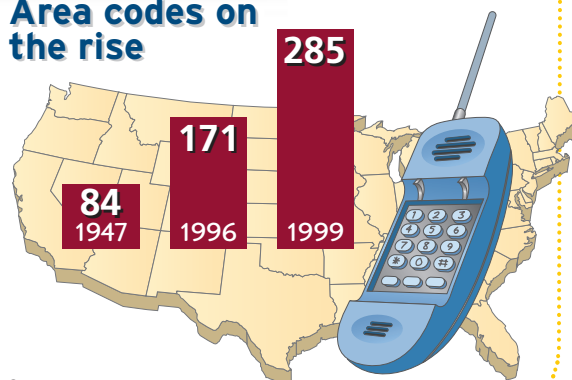
Vocabulary

- percent of change
- percent of increase
- percent of decrease

How can percents describe growth over time?

Phone companies began using area codes in 1947. The graph shows the number of area codes in use in different years. The growth in the number of area codes can be described by using a percent of change.

Area codes on the rise



Source: Associated Press

PERCENT OF CHANGE When an increase or decrease is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original, the percent of change is a **percent of decrease**.

Example 1 Find Percent of Change

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change.

a. original: 25
new: 28

Find the *amount* of change. Since the new amount is greater than the original, the percent of change is a percent of increase.

$$28 - 25 = 3$$

Find the percent using the original number, 25, as the base.

$$\begin{aligned}\frac{\text{change}}{\text{original amount}} &\rightarrow \frac{3}{25} = \frac{r}{100} \\ 3(100) &= 25(r) \\ 300 &= 25r \\ \frac{300}{25} &= \frac{25r}{25} \\ 12 &= r\end{aligned}$$

The percent of increase is 12%.

b. original: 30
new: 12

The percent of change is a percent of decrease because the new amount is less than the original. Find the change.

$$30 - 12 = 18$$

Find the percent using the original number, 30, as the base.

$$\begin{aligned}\frac{\text{change}}{\text{original amount}} &\rightarrow \frac{18}{30} = \frac{r}{100} \\ 18(100) &= 30(r) \\ 1800 &= 30r \\ \frac{1800}{30} &= \frac{30r}{30} \\ 60 &= r\end{aligned}$$

The percent of decrease is 60%.

Study Tip

Look Back

To review the **percent proportion**, see page 834.



Football •

On November 12, 1892, the Allegheny Athletic Association paid William "Pudge" Heffelfinger \$500 to play football. This game is considered the start of professional football.

Source: *World Book Encyclopedia*

Example 2 Find the Missing Value

FOOTBALL The field used by the National Football League (NFL) is 120 yards long. The length of the field used by the Canadian Football League (CFL) is 25% longer than the one used by the NFL. What is the length of the field used by the CFL?

Let ℓ = the length of the CFL field. Since 25% is a percent of increase, the length of the NFL field is less than the length of the CFL field. Therefore, $\ell - 120$ represents the amount of change.

$$\text{change} \rightarrow \frac{\ell - 120}{120} = \frac{25}{100} \quad \text{Percent proportion}$$

$$(\ell - 120)(100) = 120(25) \quad \text{Find the cross products.}$$

$$100\ell - 12,000 = 3000 \quad \text{Distributive Property}$$

$$100\ell - 12,000 + 12,000 = 3000 + 12,000 \quad \text{Add 12,000 to each side.}$$

$$100\ell = 15,000 \quad \text{Simplify.}$$

$$\frac{100\ell}{100} = \frac{15,000}{100} \quad \text{Divide each side by 100.}$$

$$\ell = 50 \quad \text{Simplify.}$$

The length of the field used by the CFL is 150 yards.

SOLVE PROBLEMS Two applications of percent of change are sales tax and discounts. Sales tax is a tax that is added to the cost of the item. It is an example of a percent of increase. Discount is the amount by which the regular price of an item is reduced. It is an example of a percent of decrease.

Example 3 Find Amount After Sales Tax

SALES TAX A concert ticket costs \$45. If the sales tax is 6.25%, what is the total price of the ticket?

The tax is 6.25% of the price of the ticket.

$$\begin{aligned} 6.25\% \text{ of } \$45 &= 0.0625 \times 45 & 6.25\% &= 0.0625 \\ &= 2.8125 & \text{Use a calculator.} \end{aligned}$$

Round \$2.8125 to \$2.82 since tax is always rounded up to the nearest cent. Add this amount to the original price.

$$\$45.00 + \$2.82 = \$47.82$$

The total price of the ticket is \$47.82.

Example 4 Find Amount After Discount

DISCOUNT A sweater is on sale for 35% off the original price. If the original price of the sweater is \$38, what is the discounted price?

The discount is 35% of the original price.

$$\begin{aligned} 35\% \text{ of } \$38 &= 0.35 \times 38 & 35\% &= 0.35 \\ &= 13.30 & \text{Use a calculator.} \end{aligned}$$

Subtract \$13.30 from the original price.

$$\$38.00 - \$13.30 = \$24.70$$

The discounted price of the sweater is \$24.70.



Check for Understanding

Concept Check

1. Compare and contrast percent of increase and percent of decrease.
2. **OPEN ENDED** Give a counterexample to the statement *The percent of change must always be less than 100%*.
3. **FIND THE ERROR** Laura and Cory are writing proportions to find the percent of change if the original number is 20 and the new number is 30.

Laura

Amount of change: $30 - 20 = 10$

$$\frac{10}{20} = \frac{r}{100}$$

Cory

Amount of change: $30 - 20 = 10$

$$\frac{10}{30} = \frac{r}{100}$$

Who is correct? Explain your reasoning.

Guided Practice

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

- | | |
|----------------------------|------------------------------|
| 4. original: 72
new: 36 | 5. original: 45
new: 50 |
| 6. original: 14
new: 16 | 7. original: 150
new: 120 |

Find the total price of each item.

- | | |
|---|--|
| 8. software: \$39.50
sales tax: 6.5% | 9. compact disc: \$15.99
sales tax: 5.75% |
|---|--|

Find the discounted price of each item.

- | | |
|-------------------------------------|------------------------------------|
| 10. jeans: \$45.00
discount: 25% | 11. book: \$19.95
discount: 33% |
|-------------------------------------|------------------------------------|

Application

EDUCATION For Exercises 12 and 13, use the following information.

According to the Census Bureau, the average income of a person with a bachelor's degree is \$40,478, and the average income of a person with a high school diploma is \$22,895.

12. Write an equation that could be used to find the percent of increase in average income for a person with a high school diploma to average income for a person with a bachelor's degree.
13. What is the percent of increase?

Practice and Apply

Homework Help

For Exercises	See Examples
14–27	1
28–30, 46, 47	2
31–36	3
37–42	4
43–45	3, 4

Extra Practice

See page 827.

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

- | | | |
|--------------------------------|---------------------------------|---------------------------------|
| 14. original: 50
new: 70 | 15. original: 25
new: 18 | 16. original: 66
new: 30 |
| 17. original: 58
new: 152 | 18. original: 13.7
new: 40.2 | 19. original: 15.6
new: 11.4 |
| 20. original: 132
new: 150 | 21. original: 85
new: 90 | 22. original: 32.5
new: 30 |
| 23. original: 9.8
new: 12.1 | 24. original: 40
new: 32.5 | 25. original: 25
new: 21.5 |



Military

A military career can involve many different duties like working in a hospital, programming computers, or repairing helicopters. The military provides training and work in these fields and others for the Army, Navy, Marine Corps, Air Force, Coast Guard, and the Air and Army National Guard.

Online Research

For information about a career in the military, visit: www.algebra1.com/careers

26. **THEME PARKS** In 1990, 253 million people visited theme parks in the United States. In 2000, the number of visitors increased to 317 million people. What was the percent of increase?
27. **MILITARY** In 1987, the United States had 2 million active-duty military personnel. By 2000, there were only 1.4 million active-duty military personnel. What was the percent of decrease?
28. The percent of increase is 16%. If the new number is 522, find the original number.
29. **FOOD** In order for a food to be marked "reduced fat," it must have at least 25% less fat than the same full-fat food. If one ounce of reduced fat chips has 6 grams of fat, what is the least amount of fat in one ounce of regular chips?
30. **TECHNOLOGY** From January, 1996, to January, 2001, the number of internet hosts increased by 1054%. There were 109.6 million internet hosts in January, 2001. Find the number of internet hosts in January, 1996.

Find the total price of each item.

- | | | |
|------------------------------------|--|------------------------------------|
| 31. umbrella: \$14.00
tax: 5.5% | 32. backpack: \$35.00
tax: 7% | 33. candle: \$7.50
tax: 5.75% |
| 34. hat: \$18.50
tax: 6.25% | 35. clock radio: \$39.99
tax: 6.75% | 36. sandals: \$29.99
tax: 5.75% |

Find the discounted price of each item.

- | | | |
|--------------------------------------|-------------------------------------|-------------------------------------|
| 37. shirt: \$45.00
discount: 40% | 38. socks: \$6.00
discount: 20% | 39. watch: \$37.55
discount: 35% |
| 40. gloves: \$24.25
discount: 33% | 41. suit: \$175.95
discount: 45% | 42. coat: \$79.99
discount: 30% |

Find the final price of each item.

- | | | |
|--|--|---|
| 43. lamp: \$120.00
discount: 20%
tax: 6% | 44. dress: \$70.00
discount: 30%
tax: 7% | 45. camera: \$58.00
discount: 25%
tax: 6.5% |
|--|--|---|

POPULATION For Exercises 46 and 47, use the following table.

Country	1997 Population (billions)	Projected Percent of Increase for 2050
China	1.24	22.6%
India	0.97	57.8%
United States	0.27	44.4%

Source: USA TODAY

46. What are the projected 2050 populations for each country in the table?
47. Which of these three countries is projected to be the most populous in 2050?
48. **RESEARCH** Use the Internet or other reference to find the tuition for the last several years at a college of your choice. Find the percent of change for the tuition during these years. Predict the tuition for the year you plan to graduate from high school.
49. **CRITICAL THINKING** Are the following expressions *sometimes*, *always*, or *never* equal? Explain your reasoning.

$x%$ of y

$y%$ of x



50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can percents describe growth over time?

Include the following in your answer:

- the percent of increase in the number of area codes from 1996 to 1999, and
- an explanation of why knowing a percent of change can be more informative than knowing how much the quantity changed.



51. The number of students at Franklin High School increased from 840 to 910 over a 5-year period. Which proportion represents the percent of change?

(A) $\frac{70}{910} = \frac{r}{100}$ (B) $\frac{70}{840} = \frac{r}{100}$ (C) $\frac{r}{910} = \frac{70}{100}$ (D) $\frac{r}{840} = \frac{70}{100}$

52. The list price of a television is \$249.00. If it is on sale for 30% off the list price, what is the sale price of the television?

(A) \$74.70 (B) \$149.40 (C) \$174.30 (D) \$219.00

Maintain Your Skills

Mixed Review

Solve each proportion. (Lesson 3-6)

53. $\frac{a}{45} = \frac{3}{15}$

54. $\frac{2}{3} = \frac{8}{d}$

55. $\frac{5.22}{13.92} = \frac{t}{48}$

Solve each equation. Then check your solution. (Lesson 3-5)

56. $6n + 3 = -3$

57. $7 + 5c = -23$

58. $18 = 4a - 2$

Find each quotient. (Lesson 2-4)

59. $\frac{2}{5} \div 4$

60. $-\frac{4}{5} \div \frac{2}{3}$

61. $-\frac{1}{9} \div \left(-\frac{3}{4}\right)$

State whether each equation is *true* or *false* for the value of the variable given. (Lesson 1-3)

62. $a^2 + 5 = 17 - a$, $a = 3$

63. $2v^2 + v = 65$, $v = 5$

64. $8y - y^2 = y + 10$, $y = 4$

65. $16p - p = 15p$, $p = 2.5$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation. Then check your solution. (To review *solving equations*, see Lesson 3-5.)

66. $-43 - 3t = 2 - 6t$

67. $7y + 7 = 3y - 5$

68. $7(d - 3) - 2 = 5$

69. $6(p + 3) = 4(p - 1)$

70. $-5 = 4 - 2(a - 5)$

71. $8x - 4 = -10x + 50$

Practice Quiz 2

Lessons 3-4 through 3-7

Solve each equation. Then check your solution. (Lessons 3-4 and 3-5)

1. $-3x - 7 = 18$

2. $5 = \frac{m - 5}{4}$

3. $4h + 5 = 11$

4. $5d - 6 = 3d + 9$

5. $7 + 2(w + 1) = 2w + 9$

6. $-8(4 + 9r) = 7(-2 - 11r)$

Solve each proportion. (Lesson 3-6)

7. $\frac{2}{10} = \frac{1}{a}$

8. $\frac{3}{5} = \frac{24}{x}$

9. $\frac{y}{4} = \frac{y + 5}{8}$

10. **POSTAGE** In 1975, the cost of a first-class stamp was 10¢. In 2001, the cost of a first-class stamp became 34¢. What is the percent of increase in the price of a stamp? (Lesson 3-7)





Reading Mathematics

Sentence Method and Proportion Method

Recall that you can solve percent problems using two different methods. With either method, it is helpful to use “clue” words such as *is* and *of*. In the sentence method, *is* means equals and *of* means multiply. With the proportion method, the “clue” words indicate where to place the numbers in the proportion.

Sentence Method

15% of 40 is what number?

$$0.15 \cdot 40 = ?$$

Proportion Method

15% of 40 is what number?

$$\frac{(\text{is}) P}{(\text{of}) B} = \frac{R(\text{percent})}{100} \rightarrow \frac{P}{40} = \frac{15}{100}$$

You can use the proportion method to solve percent of change problems. In this case, use the proportion $\frac{\text{difference}}{\text{original}} = \frac{\%}{100}$. When reading a percent of change problem, or any other word problem, look for the important numerical information.

Example In chemistry class, Kishi heated 20 milliliters of water. She let the water boil for 10 minutes. Afterward, only 17 milliliters of water remained, due to evaporation. What is the percent of decrease in the amount of water?

$$\frac{\text{difference}}{\text{original}} = \frac{\%}{100} \rightarrow \frac{20 - 17}{20} = \frac{r}{100} \quad \text{Percent proportion}$$

$$\frac{3}{20} = \frac{r}{100} \quad \text{Simplify.}$$

$$3(100) = 20(r) \quad \text{Find the cross products.}$$

$$300 = 20r \quad \text{Simplify.}$$

$$\frac{300}{20} = \frac{20r}{20} \quad \text{Divide each side by 20.}$$

$$15 = r \quad \text{Simplify.}$$

There was a 15% decrease in the amount of water.

Reading to Learn

Give the original number and the amount of change. Then write and solve a percent proportion.

1. Monsa needed to lose weight for wrestling. At the start of the season, he weighed 166 pounds. By the end of the season, he weighed 158 pounds. What is the percent of decrease in Monsa’s weight?
2. On Carla’s last Algebra test, she scored 94 points out of 100. On her first Algebra test, she scored 75 points out of 100. What is the percent of increase in her score?
3. In a catalog distribution center, workers processed an average of 12 orders per hour. After a reward incentive was offered, workers averaged 18 orders per hour. What is the percent of increase in production?

Solving Equations and Formulas

What You'll Learn

- Solve equations for given variables.
- Use formulas to solve real-world problems.

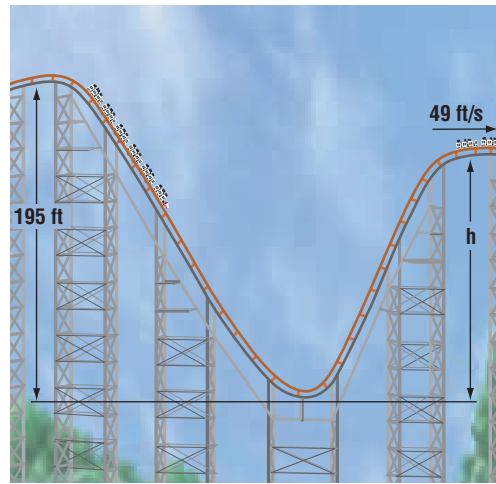
Vocabulary

- dimensional analysis

How are equations used to design roller coasters?

Ron Toomer designs roller coasters, including the Magnum XL-200. This roller coaster starts with a vertical drop of 195 feet and then ascends a second shorter hill. Suppose when designing this coaster, Mr. Toomer decided he wanted to adjust the height of the second hill so that the coaster would have a speed of 49 feet per second when it reached its top.

If we ignore friction, the equation $g(195 - h) = \frac{1}{2}v^2$ can be used to find the height of the second hill. In this equation, g represents the force of gravity (32 feet per second squared), h is the height of the second hill, and v is the velocity of the coaster when it reaches the top of the second hill.



SOLVE FOR VARIABLES Some equations such as the one above contain more than one variable. At times, you will need to solve these equations for one of the variables.

Example 1 Solve an Equation for a Specific Variable

Solve $3x - 4y = 7$ for y .

$$3x - 4y = 7$$

Original equation

$$3x - 4y - 3x = 7 - 3x$$

Subtract $3x$ from each side.

$$-4y = 7 - 3x$$

Simplify.

$$\frac{-4y}{-4} = \frac{7 - 3x}{-4}$$

Divide each side by -4 .

$$y = \frac{7 - 3x}{-4} \text{ or } \frac{3x - 7}{4}$$

Simplify.

The value of y is $\frac{3x - 7}{4}$.

It is sometimes helpful to use the Distributive Property to isolate the variable for which you are solving an equation or formula.

Example 2 Solve an Equation for a Specific Variable

Solve $2m - t = sm + 5$ for m .

$$2m - t = sm + 5 \quad \text{Original equation}$$

$$2m - t - sm = sm + 5 - sm \quad \text{Subtract } sm \text{ from each side.}$$

$$2m - t - sm = 5 \quad \text{Simplify.}$$

$$2m - t - sm + t = 5 + t \quad \text{Add } t \text{ to each side.}$$

$$2m - sm = 5 + t \quad \text{Simplify.}$$

$$m(2 - s) = 5 + t \quad \text{Use the Distributive Property.}$$

$$\frac{m(2 - s)}{2 - s} = \frac{5 + t}{2 - s} \quad \text{Divide each side by } 2 - s.$$

$$m = \frac{5 + t}{2 - s} \quad \text{Simplify.}$$

The value of m is $\frac{5 + t}{2 - s}$. Since division by 0 is undefined, $2 - s \neq 0$ or $s \neq 2$.

USE FORMULAS Many real-world problems require the use of formulas. Sometimes solving a formula for a specific variable will help you solve the problem.

Example 3 Use a Formula to Solve Problems

WEATHER Use the information about the Kansas City hailstorm at the left. The formula for the circumference of a circle is $C = 2\pi r$, where C represents circumference and r represent radius.

a. Solve the formula for r .

$$C = 2\pi r \quad \text{Formula for circumference}$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \quad \text{Divide each side by } 2\pi.$$

$$\frac{C}{2\pi} = r \quad \text{Simplify.}$$

b. Find the radius of one of the large hailstones that fell on Kansas City in 1898.

$$\frac{C}{2\pi} = r \quad \text{Formula for radius}$$

$$\frac{9.5}{2\pi} = r \quad C = 9.5$$

$$1.5 \approx r \quad \text{The largest hailstones had a radius of about 1.5 inches.}$$

More About . . .



Weather

On May 14, 1898, a severe hailstorm hit Kansas City. The largest hailstones were 9.5 inches in circumference. Windows were broken in nearly every house in the area.

Source: National Weather Service

When using formulas, you may want to use dimensional analysis. **Dimensional analysis** is the process of carrying units throughout a computation.

Example 4 Use Dimensional Analysis

PHYSICAL SCIENCE The formula $s = \frac{1}{2}at^2$ represents the distance s that a free-falling object will fall near a planet or the moon in a given time t . In the formula, a represents the acceleration due to gravity.

a. Solve the formula for a .

$$s = \frac{1}{2}at^2 \quad \text{Original formula}$$

$$\frac{2}{t^2}(s) = \frac{2}{t^2}\left(\frac{1}{2}at^2\right) \quad \text{Multiply each side by } \frac{2}{t^2}.$$

$$\frac{2s}{t^2} = a \quad \text{Simplify.}$$



- b. A free-falling object near the moon drops 20.5 meters in 5 seconds. What is the value of a for the moon?

$$a = \frac{2s}{t^2} \quad \text{Formula for } a$$

$$a = \frac{2(20.5m)}{(5s)^2} \quad s = 20.5m \text{ and } t = 5s.$$

$$a = \frac{1.64m}{s^2} \text{ or } 1.64 \text{ m/s}^2 \quad \text{Use a calculator.}$$

The acceleration due to gravity on the moon is 1.64 meters per second squared.

Check for Understanding

Concept Check

- List the steps you would use to solve $ax - y = az + w$ for a .
- Describe the possible values of t if $s = \frac{r}{t-2}$.
- OPEN ENDED** Write a formula for A , the area of a geometric figure such as a triangle or rectangle. Then solve the formula for a variable other than A .

Guided Practice

Solve each equation or formula for the variable specified.

4. $-3x + b = 6x$, for x

5. $-5a + y = -54$, for a

6. $4z + b = 2z + c$, for z

7. $\frac{y+a}{3} = c$, for y

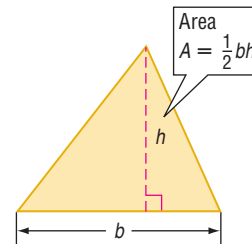
8. $p = a(b+c)$, for a

9. $mw - t = 2w + 5$, for w

Application

GEOMETRY For Exercises 10–12, use the formula for the area of a triangle.

- Find the area of a triangle with a base of 18 feet and a height of 7 feet.
- Solve the formula for h .
- What is the height of a triangle with area of 28 square feet and base of 8 feet?



Practice and Apply

Homework Help

For Exercises	See Examples
13–30	1, 2
31–41	3, 4

Extra Practice

See page 827.

Solve each equation or formula for the variable specified.

13. $5g + h = g$, for g

14. $8t - r = 12t$, for t

15. $y = mx + b$, for m

16. $v = r + at$, for a

17. $3y + z = am - 4y$, for y

18. $9a - 2b = c + 4a$, for a

19. $km + 5x = 6y$, for m

20. $4b - 5 = -t$, for b

21. $\frac{3ax - n}{5} = -4$, for x

22. $\frac{5x + y}{a} = 2$, for a

23. $\frac{by + 2}{3} = c$, for y

24. $\frac{6c - t}{7} = b$, for c

25. $c = \frac{3}{4}y + b$, for y

26. $\frac{3}{5}m + a = b$, for m

27. $S = \frac{n}{2}(A + t)$, for A

28. $p(t + 1) = -2$, for t

29. $at + b = ar - c$, for a

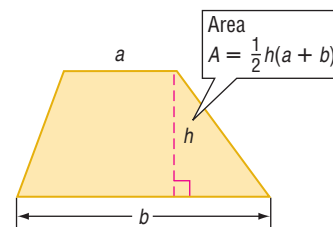
30. $2g - m = 5 - gh$, for g

Write an equation and solve for the variable specified.

31. Five less than a number t equals another number r plus six. Solve for t .
32. Five minus twice a number p equals six times another number q plus one. Solve for p .
33. Five eighths of a number x is three more than one half of another number y . Solve for y .

GEOMETRY For Exercises 34 and 35, use the formula for the area of a trapezoid.

34. Solve the formula for h .
35. What is the height of a trapezoid with an area of 60 square meters and bases of 8 meters and 12 meters?



WORK For Exercises 36 and 37, use the following information.

The formula $s = \frac{w - 10e}{m}$ is often used by placement services to find keyboarding speeds. In the formula, s represents the speed in words per minute, w represents the number of words typed, e represents the number of errors, and m represents the number of minutes typed.

36. Solve the formula for e .
37. If Miguel typed 410 words in 5 minutes and received a keyboard speed of 76 words per minute, how many errors did he make?

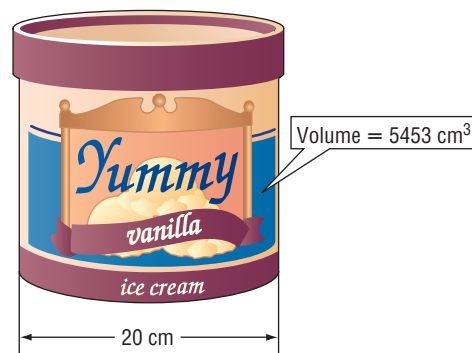
FLOORING For Exercises 38 and 39, use the following information.

The formula $P = \frac{1.2W}{H^2}$ represents the amount of pressure exerted on the floor by the heel of a shoe. In this formula, P represents the pressure in pounds per square inch, W represents the weight of a person wearing the shoe in pounds, and H is the width of the heel of the shoe in inches.

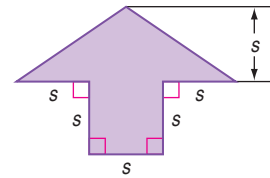
38. Solve the formula for W .
39. Find the weight of the person if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch.

40. **ROCKETRY** In the book *October Sky*, high school students were experimenting with different rocket designs. One formula they used was $R = \frac{S + F + P}{S + P}$, which relates the mass ratio R of a rocket to the mass of the structure S , the mass of the fuel F , and the mass of the payload P . The students needed to determine how much fuel to load in the rocket. How much fuel should be loaded in a rocket whose basic structure and payload each have a mass of 900 grams, if the mass ratio is to be 6?

41. **PACKAGING** The Yummy Ice Cream Company wants to package ice cream in cylindrical containers that have a volume of 5453 cubic centimeters. The marketing department decides the diameter of the base of the containers should be 20 centimeters. How tall should the containers be?
(Hint: $V = \pi r^2 h$)



42. **CRITICAL THINKING** Write a formula for the area of the arrow.



43. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

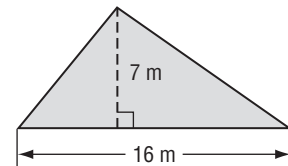
How are equations used to design roller coasters?

Include the following in your answer:

- a list of steps you could use to solve the equation for h , and
- the height of the second hill of the roller coaster.

Standardized Test Practice

44. If $2x + y = 5$, what is the value of $4x$?
- (A) $10 - y$ (B) $10 - 2y$
 (C) $\frac{5 - y}{2}$ (D) $\frac{10 - y}{2}$
45. What is the area of the triangle?
- (A) 23 m^2 (B) 28 m^2
 (C) 56 m^2 (D) 112 m^2



Maintain Your Skills

Mixed Review

Find the discounted price of each item. (Lesson 3-7)

46. camera: \$85.00 47. scarf: \$15.00 48. television: \$299.00
 discount: 20% discount: 35% discount: 15%

Solve each proportion. (Lesson 3-6)

49. $\frac{2}{9} = \frac{5}{a}$ 50. $\frac{15}{32} = \frac{t}{8}$ 51. $\frac{x+1}{8} = \frac{3}{4}$

Write the numbers in each set in order from least to greatest. (Lesson 2-7)

52. $\frac{1}{4}, \sqrt{\frac{1}{4}}, 0.\bar{5}, 0.2$ 53. $\sqrt{5}, 3, \frac{2}{3}, 1.1$

Find each sum or difference. (Lesson 2-2)

54. $2.18 + (-5.62)$ 55. $-\frac{1}{2} - \left(-\frac{3}{4}\right)$ 56. $-\frac{2}{3} - \frac{2}{5}$

Name the property illustrated by each statement. (Lesson 1-4)

57. $mnp = 1mnp$ 58. If $6 = 9 - 3$, then $9 - 3 = 6$.
 59. $32 + 21 = 32 + 21$ 60. $8 + (3 + 9) = 8 + 12$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Use the Distributive Property to rewrite each expression without parentheses. (To review the Distributive Property, see Lesson 1-5.)

61. $6(2 - t)$ 62. $(5 + 2m)3$ 63. $-7(3a + b)$
 64. $\frac{2}{3}(6h - 9)$ 65. $-\frac{3}{5}(15 - 5t)$ 66. $0.25(6p + 12)$



3-9

Weighted Averages

What You'll Learn

- Solve mixture problems.
- Solve uniform motion problems.

Vocabulary

- weighted average
- mixture problem
- uniform motion problem

How are scores calculated in a figure skating competition?

In individual figure skating competitions, the score for the long program is worth twice the score for the short program. Suppose Olympic gold medal winner Ilia Kulik scores 5.5 in the short program and 5.8 in the long program at a competition. His final score is determined using a weighted average.

$$\begin{aligned} \frac{5.5(1) + 5.8(2)}{1 + 2} &= \frac{5.5 + 11.6}{3} \\ &= \frac{17.1}{3} \text{ or } 5.7 \end{aligned} \quad \text{His final score would be 5.7.}$$

MIXTURE PROBLEMS Ilia Kulik's average score is an example of a weighted average. The **weighted average** M of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

Mixture problems are problems in which two or more parts are combined into a whole. They are solved using weighted averages.

Example 1 Solve a Mixture Problem with Prices

TRAIL MIX Assorted dried fruit sells for \$5.50 per pound. How many pounds of mixed nuts selling for \$4.75 per pound should be mixed with 10 pounds of dried fruit to obtain a trail mix that sells for \$4.95 per pound?

Let w = the number of pounds of mixed nuts in the mixture. Make a table.

	Units (lb)	Price per Unit (lb)	Total Price
Dried Fruit	10	\$5.50	$5.50(10)$
Mixed Nuts	w	\$4.75	$4.75w$
Trail Mix	$10 + w$	\$4.95	$4.95(10 + w)$

$$\underbrace{\text{Price of dried fruit}}_{5.50(10)} \quad \text{plus} \quad \underbrace{\text{price of nuts}}_{4.75w} \quad \text{equals} \quad \underbrace{\text{price of trail mix.}}_{4.95(10 + w)}$$

$$5.50(10) + 4.75w = 4.95(10 + w) \quad \text{Original equation}$$

$$55.00 + 4.75w = 49.50 + 4.95w \quad \text{Distributive Property}$$

$$55.00 + 4.75w - 4.75w = 49.50 + 4.95w - 4.75w \quad \text{Subtract } 4.75w \text{ from each side.}$$

$$55.00 = 49.50 + 0.20w \quad \text{Simplify.}$$

$$55.00 - 49.50 = 49.50 + 0.20w - 49.50 \quad \text{Subtract } 49.50 \text{ from each side.}$$

$$5.50 = 0.20w \quad \text{Simplify.}$$

$$\frac{5.50}{0.20} = \frac{0.20w}{0.20} \quad \text{Divide each side by } 0.20.$$

$$27.5 = w \quad \text{Simplify.}$$

27.5 pounds of nuts should be mixed with 10 pounds of dried fruit.

Sometimes mixture problems are expressed in terms of percents.

Example 2 Solve a Mixture Problem with Percents

SCIENCE A chemistry experiment calls for a 30% solution of copper sulfate. Kendra has 40 milliliters of 25% solution. How many milliliters of 60% solution should she add to obtain the required 30% solution?

Let x = the amount of 60% solution to be added. Make a table.

	Amount of Solution (mL)	Amount of Copper Sulfate
25% Solution	40	$0.25(40)$
60% Solution	x	$0.60x$
30% Solution	$40 + x$	$0.30(40 + x)$

Write and solve an equation using the information in the table.

$$\underbrace{\text{Amount of copper sulfate in 25\% solution}}_{0.25(40)} \quad \underbrace{\text{plus}}_{+} \quad \underbrace{\text{amount of copper sulfate in 60\% solution}}_{0.60x} \quad \underbrace{\text{equals}}_{=} \quad \underbrace{\text{amount of copper sulfate in 30\% solution.}}_{0.30(40 + x)}$$

$$0.25(40) + 0.60x = 0.30(40 + x)$$

Original equation

$$10 + 0.60x = 12 + 0.30x$$

Distributive Property

$$10 + 0.60x - 0.30x = 12 + 0.30x - 0.30x$$

Subtract $0.30x$ from each side.

$$10 + 0.30x = 12$$

Simplify.

$$10 + 0.30x - 10 = 12 - 10$$

Subtract 10 from each side.

$$0.30x = 2$$

Simplify.

$$\frac{0.30x}{0.30} = \frac{2}{0.30}$$

Divide each side by 0.30.

$$x \approx 6.67$$

Simplify.

Kendra should add 6.67 milliliters of the 60% solution to the 40 milliliters of the 25% solution.

Study Tip

Mixture Problems

When you organize the information in mixture problems, remember that the final mixture must contain the sum of the parts in the correct quantities and at the correct percents.

UNIFORM MOTION PROBLEMS Motion problems are another application of weighted averages. **Uniform motion problems** are problems where an object moves at a certain speed, or rate. The formula $d = rt$ is used to solve these problems. In the formula, d represents distance, r represents rate, and t represents time.

Example 3 Solve for Average Speed

TRAVEL On Alberto's drive to his aunt's house, the traffic was light, and he drove the 45-mile trip in one hour. However, the return trip took him two hours. What was his average speed for the round trip?

To find the average speed for each leg of the trip, rewrite $d = rt$ as $r = \frac{d}{t}$.

Going

$$r = \frac{d}{t}$$

$$= \frac{45 \text{ miles}}{1 \text{ hour}} \text{ or } 45 \text{ miles per hour}$$

Returning

$$r = \frac{d}{t}$$

$$= \frac{45 \text{ miles}}{2 \text{ hours}} \text{ or } 22.5 \text{ miles per hour}$$

You may think that the average speed of the trip would be $\frac{45 + 22.5}{2}$ or 33.75 miles per hour. However, Alberto did not drive at these speeds for equal amounts of time. You must find the weighted average for the trip.

Round Trip

$$M = \frac{45(1) + 22.5(2)}{1 + 2} \quad \text{Definition of weighted average}$$

$$= \frac{90}{3} \text{ or } 30 \quad \text{Simplify.}$$

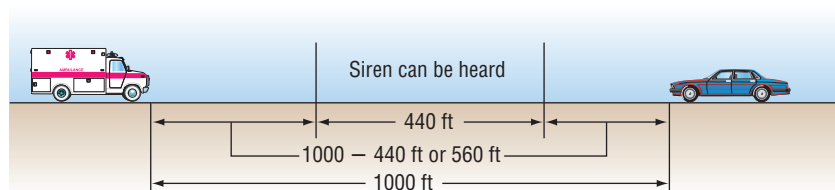
Alberto's average speed was 30 miles per hour.

Sometimes a table is useful in solving uniform motion problems.

Example 4 Solve a Problem Involving Speeds of Two Vehicles

SAFETY Use the information about sirens at the left. A car and an emergency vehicle are heading toward each other. The car is traveling at a speed of 30 miles per hour or about 44 feet per second. The emergency vehicle is traveling at a speed of 50 miles per hour or about 74 feet per second. If the vehicles are 1000 feet apart and the conditions are ideal, in how many seconds will the driver of the car first hear the siren?

Draw a diagram. The driver can hear the siren when the total distance traveled by the two vehicles equals $1000 - 440$ or 560 feet.



Let t = the number of seconds until the driver can hear the siren. Make a table of the information.

	r	t	$d = rt$
Car	44	t	$44t$
Emergency Squad	74	t	$74t$

Write an equation.

$$\underbrace{44t}_{\text{Distance traveled by car}} \quad \text{plus} \quad \underbrace{74t}_{\text{distance traveled by emergency vehicle}} \quad \text{equals} \quad \underbrace{560}_{\text{560 feet.}}$$

Solve the equation.

$$44t + 74t = 560 \quad \text{Original equation}$$

$$118t = 560 \quad \text{Simplify.}$$

$$\frac{118t}{118} = \frac{560}{118} \quad \text{Divide each side by 118.}$$

$$t \approx 4.75 \quad \text{Round to the nearest hundredth.}$$

The driver of the car will hear the siren in about 4.75 seconds.



Safety

Under ideal conditions, a siren can be heard from up to 440 feet. However, under normal conditions, a siren can be heard from only 125 feet.

Source: U.S. Department of Transportation

Check for Understanding

Concept Check

- OPEN ENDED** Give a real-world example of a weighted average.
- Write** the formula used to solve uniform motion problems and tell what each letter represents.
- Make a table** that can be used to solve the following problem.
Lakeisha has \$2.55 in dimes and quarters. She has 8 more dimes than quarters. How many quarters does she have?

Guided Practice

FOOD For Exercises 4–7, use the following information.

How many quarts of pure orange juice should Michael add to a 10% orange drink to create 6 quarts of a 40% orange juice mixture? Let p represent the number of quarts of pure orange juice he should add to the orange drink.

- Copy and complete the table representing the problem.

	Quarts	Amount of Orange Juice
10% Juice	$6 - p$	
100% Juice	p	
40% Juice		

- Write an equation to represent the problem.
- How much pure orange juice should Michael use?
- How much 10% juice should Michael use?
- BUSINESS** The Nut Shoppe sells walnuts for \$4.00 a pound and cashews for \$7.00 a pound. How many pounds of cashews should be mixed with 10 pounds of walnuts to obtain a mixture that sells for \$5.50 a pound?
- GRADES** Many schools base a student's grade point average, or GPA, on the student's grade and the class credit rating. Brittany's grade card for this semester is shown. Find Brittany's GPA if a grade of A equals 4 and a B equals 3.
- CYCLING** Two cyclists begin traveling in the same direction on the same bike path. One travels at 20 miles per hour, and the other travels at 14 miles per hour. When will the cyclists be 15 miles apart?

Class	Credit Rating	Grade
Algebra 1	1	A
Science	1	B
English	1	A
Spanish	1	B
Phys. Ed.	$\frac{1}{2}$	A

Practice and Apply

Homework Help

For Exercises	See Examples
11–18, 22–25, 27–29, 33	1, 2
19–21, 26, 30–32, 34	3, 4

Extra Practice

See page 828.

BUSINESS For Exercises 11–14, use the following information.

Cookies Inc. sells peanut butter cookies for \$6.50 per dozen and chocolate chip cookies for \$9.00 per dozen. Yesterday, they sold 85 dozen more peanut butter cookies than chocolate chip cookies. The total sales for both types of cookies were \$4055.50. Let p represent the number of dozens of peanut butter cookies sold.

- Copy and complete the table representing the problem.

	Number of Dozens	Price per Dozen	Total Price
Peanut Butter Cookies	p		
Chocolate Chip Cookies	$p - 85$		

- Write an equation to represent the problem.
- How many dozen peanut butter cookies were sold?
- How many dozen chocolate chip cookies were sold?

METALS For Exercises 15–18, use the following information.

In 2000, the international price of gold was \$270 per ounce, and the international price of silver was \$5 per ounce. Suppose gold and silver were mixed to obtain 15 ounces of an alloy worth \$164 per ounce. Let g represent the amount of gold used in the alloy.

15. Copy and complete the table representing the problem.

	Number of Ounces	Price per Ounce	Value
Gold	g		
Silver	$15 - g$		
Alloy			

16. Write an equation to represent the problem.
17. How much gold was used in the alloy?
18. How much silver was used in the alloy?

TRAVEL For Exercises 19–21, use the following information.

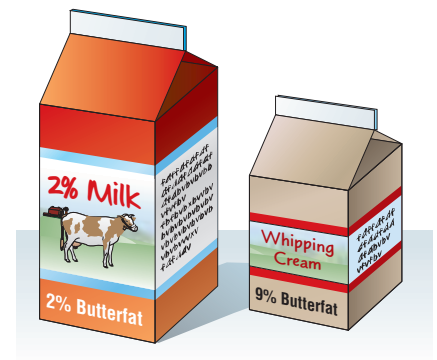
Two trains leave Pittsburgh at the same time, one traveling east and the other traveling west. The eastbound train travels at 40 miles per hour, and the westbound train travels at 30 miles per hour. Let t represent the amount of time since their departure.

19. Copy and complete the table representing the situation.

	r	t	$d = rt$
Eastbound Train			
Westbound Train			

20. Write an equation that could be used to determine when the trains will be 245 miles apart.
21. In how many hours will the trains be 245 miles apart?
22. **FUND-RAISING** The Madison High School marching band sold gift wrap. The gift wrap in solid colors sold for \$4.00 per roll, and the print gift wrap sold for \$6.00 per roll. The total number of rolls sold was 480, and the total amount of money collected was \$2340. How many rolls of each kind of gift wrap were sold?
23. **COFFEE** Charley Baroni owns a specialty coffee store. He wants to create a special mix using two coffees, one priced at \$6.40 per pound and the other priced at \$7.28 per pound. How many pounds of the \$7.28 coffee should he mix with 9 pounds of the \$6.40 coffee to sell the mixture for \$6.95 per pound?

24. **FOOD** Refer to the graphic at the right. How much whipping cream and 2% milk should be mixed to obtain 35 gallons of milk with 4% butterfat?

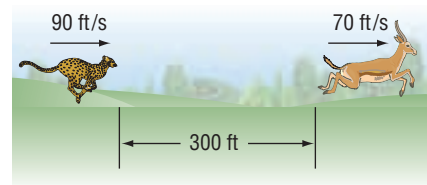


25. **METALS** An alloy of metals is 25% copper. Another alloy is 50% copper. How much of each alloy should be used to make 1000 grams of an alloy that is 45% copper?
26. **TRAVEL** An airplane flies 1000 miles due east in 2 hours and 1000 miles due south in 3 hours. What is the average speed of the airplane?



27. **SCIENCE** Hector is performing a chemistry experiment that requires 140 milliliters of a 30% copper sulfate solution. He has a 25% copper sulfate solution and a 60% copper sulfate solution. How many milliliters of each solution should he mix to obtain the needed solution?
28. **CAR MAINTENANCE** One type of antifreeze is 40% glycol, and another type of antifreeze is 60% glycol. How much of each kind should be used to make 100 gallons of antifreeze that is 48% glycol?
29. **GRADES** In Ms. Martinez's science class, a test is worth three times as much as a quiz. If a student has test grades of 85 and 92 and quiz grades of 82, 75, and 95, what is the student's average grade?
30. **RESCUE** A fishing trawler has radioed the Coast Guard for a helicopter to pick up an injured crew member. At the time of the emergency message, the trawler is 660 kilometers from the helicopter and heading toward it. The average speed of the trawler is 30 kilometers per hour, and the average speed of the helicopter is 300 kilometers per hour. How long will it take the helicopter to reach the trawler?

31. **ANIMALS** A cheetah is 300 feet from its prey. It starts to sprint toward its prey at 90 feet per second. At the same time, the prey starts to sprint at 70 feet per second. When will the cheetah catch its prey?



32. **TRACK AND FIELD** A sprinter has a bad start, and his opponent is able to start 1 second before him. If the sprinter averages 8.2 meters per second and his opponent averages 8 meters per second, will he be able to catch his opponent before the end of the 200-meter race? Explain.
33. **CAR MAINTENANCE** A car radiator has a capacity of 16 quarts and is filled with a 25% antifreeze solution. How much must be drained off and replaced with pure antifreeze to obtain a 40% antifreeze solution?
34. **TRAVEL** An express train travels 80 kilometers per hour from Ironton to Wildwood. A local train, traveling at 48 kilometers per hour, takes 2 hours longer for the same trip. How far apart are Ironton and Wildwood?
35. **FOOTBALL** NFL quarterbacks are rated for their passing performance by a type of weighted average as described in the formula below.

$$R = [50 + 2000(C \div A) + 8000(T \div A) - 10,000(I \div A) + 100(Y \div A)] \div 24$$

In this formula,

- R represents the rating,
- C represents number of completions,
- A represents the number of passing attempts,
- T represents the number to touchdown passes,
- I represents the number of interceptions, and
- Y represents the number of yards gained by passing.

In the 2000 season, Daunte Culpepper had 297 completions, 474 passing attempts, 33 touchdown passes, 16 interceptions, and 3937 passing yards. What was his rating for that year?



Online Research Data Update What is the current passing rating for your favorite quarterback? Visit www.algebra1.com/data_update to get statistics on quarterbacks.

36. **CRITICAL THINKING** Write a mixture problem for the equation $1.00x + 0.28(40) = 0.40(x + 40)$.

37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are scores calculated in a figure skating competition?

Include the following in your answer:

- an explanation of how a weighted average can be used to find a skating score, and
- a demonstration of how to find the weighted average of a skater who received a 4.9 in the short program and a 5.2 in the long program.



38. Eula Jones is investing \$6000 in two accounts, part at 4.5% and the remainder at 6%. If d represents the number of dollars invested at 4.5%, which expression represents the amount of interest earned in the account paying 6%?
- (A) $0.06d$ (B) $0.06(d - 6000)$
 (C) $0.06(d + 6000)$ (D) $0.06(6000 - d)$
39. Todd drove from Boston to Cleveland, a distance of 616 miles. His breaks, gasoline, and food stops took 2 hours. If his trip took 16 hours altogether, what was his average speed?
- (A) 38.5 mph (B) 40 mph (C) 44 mph (D) 47.5 mph

Maintain Your Skills

Mixed Review Solve each equation for the variable specified. (Lesson 3-8)

40. $3t - 4 = 6t - s$, for t

41. $a + 6 = \frac{b-1}{4}$, for b

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.

(Lesson 3-7)

42. original: 25
new: 14

43. original: 35
new: 42

44. original: 244
new: 300

45. If the probability that an event will occur is $\frac{2}{3}$, what are the odds that the event will occur? (Lesson 2-6)

Simplify each expression. (Lesson 2-3)

46. $(2b)(-3a)$

47. $3x(-3y) + (-6x)(-2y)$

48. $5s(-6t) + 2s(-8t)$

Name the set of numbers graphed. (Lesson 2-1)



WebQuest Internet Project

Can You Fit 100 Candles on a Cake?

It's time to complete your project. Use the information and data you have gathered about living to be 100 to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.



www.algebra1.com/webquest



Spreadsheet Investigation

A Follow-Up of Lesson 3-9

Finding a Weighted Average

You can use a computer spreadsheet program to calculate weighted averages. A spreadsheet allows you to make calculations and print almost anything that can be organized in a table.

The basic unit in a spreadsheet is called a **cell**. A cell may contain numbers, words, or a formula. Each cell is named by the column and row that describe its location. For example, cell B4 is in column B, row 4.

Example

Greta Norris manages the Java Roaster Coffee Shop. She has entered the price per pound and the number of pounds sold in October for each type of coffee in a spreadsheet. What was the average price per pound of coffee sold?

	A	B	C	D
1	Product	Price per Pound	Pounds Sold	Income
2	Hawaiian Cafe	16.95	59	=B2*C2
3	Mocha Java	12.59	85	=B3*C3
4	House Blend	10.75	114	=B4*C4
5	Decaf Espresso	10.15	75	=B5*C5
6	Breakfast Blend	11.25	93	=B6*C6
7	Italian Roast	9.95	55	=B7*C7
8	Total		=SUM(C2:C7)	=SUM(D2:D7)
9	Weighted Average	=D8 / C8		

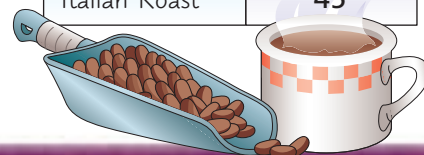
The spreadsheet shows the formula that will calculate the weighted average. The formula multiplies the price of each product by its volume and calculates its sum for all the products. Then it divides that value by the sum of the volume for all products together. To the nearest cent, the weighted average of a pound of coffee is \$11.75.

Exercises

For Exercises 1–4, use the spreadsheet of coffee prices.

1. What is the average price of a pound of coffee for the November sales shown in the table at the right?
2. How does the November weighted average change if all of the coffee prices are increased by \$1.00?
3. How does the November weighted average change if all of the coffee prices are increased by 10%?
4. Find the weighted average of a pound of coffee if the shop sold 50 pounds of each type of coffee. How does the weighted average compare to the average of the per-pound coffee prices? Explain.

Product	Pounds Sold
Hawaiian Cafe	56
Mocha Java	97
House Blend	124
Decaf Espresso	71
Breakfast Blend	69
Italian Roast	45



Vocabulary and Concept Check

Addition Property of Equality (p. 128)	identity (p. 150)	proportion (p. 155)
consecutive integers (p. 144)	means (p. 156)	rate (p. 157)
defining a variable (p. 121)	mixture problem (p. 171)	ratio (p. 155)
dimensional analysis (p. 167)	Multiplication Property of Equality (p. 135)	scale (p. 157)
Division Property of Equality (p. 137)	multi-step equations (p. 143)	solve an equation (p. 129)
equivalent equation (p. 129)	number theory (p. 144)	Subtraction Property of Equality (p. 129)
extremes (p. 156)	percent of change (p. 160)	uniform motion problem (p. 172)
formula (p. 122)	percent of decrease (p. 160)	weighted average (p. 171)
four-step problem-solving plan (p. 121)	percent of increase (p. 160)	work backward (p. 142)

Choose the correct term to complete each sentence.

1. According to the (*Addition, Multiplication*) Property of Equality, if $a = b$, then $a + c = b + c$.
2. A (*means, ratio*) is a comparison of two numbers by division.
3. A rate is the ratio of two measurements with (*the same, different*) units of measure.
4. The first step in the four-step problem-solving plan is to (*explore, solve*) the problem.
5. $2x + 1 = 2x + 1$ is an example of a(n) (*identity, formula*).
6. An equivalent equation for $3x + 5 = 7$ is ($3x = 2$, $3x = 12$).
7. If the original amount was 80 and the new amount is 90, then the percent of (*decrease, increase*) is 12.5%.
8. (*Defining the variable, Dimensional analysis*) is the process of carrying units throughout a computation.
9. The (*weighted average, rate*) of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights.
10. An example of consecutive integers is (*8 and 9, 8 and 10*).

Lesson-by-Lesson Review

3-1 Writing Equations

See pages
120–126.

Concept Summary

- Variables are used to represent unknowns when writing equations.
- Formulas given in sentence form can be written as algebraic equations.

Example

Translate the following sentence into an equation.

The sum of x and y equals 2 plus two times the product of x and y .

$$\underbrace{\text{The sum of } x \text{ and } y}_{x + y} \quad \underbrace{\text{equals}}_{=} \quad \underbrace{2}_{2} \quad \underbrace{\text{plus}}_{+} \quad \underbrace{\text{two times the product of } x \text{ and } y}_{2xy}$$

The equation is $x + y = 2 + 2xy$.



Exercises Translate each sentence into an equation. See Example 1 on page 120.

11. Three times a number n decreased by 21 is 57.
12. Four minus three times z is equal to z decreased by 2.
13. The sum of the square of a and the cube of b is 16.
14. Translate the equation $16 - 9r = r$ into a verbal sentence. See Example 4 on pages 122 and 123.

3-2 Solving Equations by Using Addition and Subtraction

See pages 128–134.

Concept Summary

- **Addition Property of Equality** For any numbers a , b , and c , if $a = b$, then $a + c = b + c$.
- **Subtraction Property of Equality** For any numbers a , b , and c , if $a = b$, then $a - c = b - c$.

Example

Solve $x - 13 = 45$. Then check your solution.

$$\begin{aligned} x - 13 &= 45 && \text{Original equation} \\ x - 13 + 13 &= 45 + 13 && \text{Add 13 to each side.} \\ x &= 58 && \text{Simplify.} \end{aligned}$$

CHECK

$$\begin{aligned} x - 13 &= 45 && \text{Original equation} \\ 58 - 13 &\stackrel{?}{=} 45 && \text{Substitute 58 for } x. \\ 45 &= 45 \checkmark && \text{Simplify.} \end{aligned} \qquad \text{The solution is 58.}$$

Exercises Solve each equation. Then check your solution.

See Examples 1–4 on pages 129 and 130.

- | | | |
|---------------------|-------------------------|--|
| 15. $r - 21 = -37$ | 16. $14 + c = -5$ | 17. $27 = 6 + p$ |
| 18. $b + (-14) = 6$ | 19. $d - (-1.2) = -7.3$ | 20. $r + \left(-\frac{1}{2}\right) = -\frac{3}{4}$ |

3-3 Solving Equations by Using Multiplication and Division

See pages 135–140.

Concept Summary

- **Multiplication Property of Equality** For any numbers a , b , and c , if $a = b$, then $ac = bc$.
- **Division Property of Equality** For any numbers a , b , and c , with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Example

Solve $\frac{4}{9}t = -72$.

$$\begin{aligned} \frac{4}{9}t &= -72 && \text{Original equation} \\ \frac{9}{4}\left(\frac{4}{9}t\right) &= \frac{9}{4}(-72) && \text{Multiply each side by } \frac{9}{4}. \\ t &= -162 && \text{Simplify.} \end{aligned}$$

CHECK

$$\begin{aligned} \frac{4}{9}t &= -72 && \text{Original equation} \\ \frac{4}{9}(-162) &\stackrel{?}{=} -72 && \text{Substitute } -162 \text{ for } t. \\ -72 &= -72 \checkmark && \text{Simplify.} \end{aligned}$$

The solution is -162 .



Exercises Solve each equation. Then check your solution.

See Examples 1–3 on pages 135 and 136.

21. $6x = -42$

22. $-7w = -49$

23. $\frac{3}{4}n = 30$

24. $-\frac{3}{5}y = -50$

25. $\frac{5}{2}a = -25$

26. $5 = \frac{r}{2}$

3-4 Solving Multi-Step Equations

See pages
142–148.

Concept Summary

- Multi-step equations can be solved by undoing the operations in reverse of the order of operations.

Example

Solve $34 = 8 - 2t$. Then check your solution.

$$34 = 8 - 2t \quad \text{Original equation}$$

$$34 - 8 = 8 - 2t - 8 \quad \text{Subtract 8 from each side.}$$

$$26 = -2t \quad \text{Simplify.}$$

$$\frac{26}{-2} = \frac{-2t}{-2} \quad \text{Divide each side by } -2.$$

$$-13 = t \quad \text{Simplify.}$$

CHECK $34 = 8 - 2t$ Original equation

$$34 \stackrel{?}{=} 8 - 2(-13) \quad \text{Substitute } -13 \text{ for } t.$$

$$34 = 34 \quad \checkmark \quad \text{The solution is } -13.$$

Exercises Solve each equation. Then check your solution.

See Examples 2–4 on page 143.

27. $4p - 7 = 5$

28. $6 = 4v + 2$

29. $\frac{y}{3} + 6 = -45$

30. $\frac{c}{-4} - 8 = -42$

31. $\frac{4d + 5}{7} = 7$

32. $\frac{7n + (-1)}{8} = 8$

3-5 Solving Equations with the Variable on Each Side

See pages
149–154.

Concept Summary

Steps for Solving Equations

Step 1 Use the Distributive Property to remove the grouping symbols.

Step 2 Simplify the expressions on each side of the equals sign.

Step 3 Use the Addition and/or Subtraction Properties of Equality to get the variables on one side of the equals sign and the numbers without variables on the other side of the equals sign.

Step 4 Simplify the expressions on each side of the equals sign.

Step 5 Use the Multiplication and/or Division Properties of Equalities to solve.



Example

Solve $\frac{3}{4}q - 8 = \frac{1}{4}q + 9$.

$$\frac{3}{4}q - 8 = \frac{1}{4}q + 9 \quad \text{Original equation}$$

$$\frac{3}{4}q - 8 - \frac{1}{4}q = \frac{1}{4}q + 9 - \frac{1}{4}q \quad \text{Subtract } \frac{1}{4}q \text{ from each side.}$$

$$\frac{1}{2}q - 8 = 9 \quad \text{Simplify.}$$

$$\frac{1}{2}q - 8 + 8 = 9 + 8 \quad \text{Add 8 to each side.}$$

$$\frac{1}{2}q = 17 \quad \text{Simplify.}$$

$$2\left(\frac{1}{2}q\right) = 2(17) \quad \text{Multiply each side by 2.}$$

$$q = 34 \quad \text{Simplify.}$$

The solution is 34.

Exercises Solve each equation. Then check your solution.

See Examples 1–4 on pages 149 and 150.

- | | | |
|-------------------------------------|---------------------------|---|
| 33. $n - 2 = 4 - 2n$ | 34. $3t - 2(t + 3) = t$ | 35. $3 - \frac{5}{6}y = 2 + \frac{1}{6}y$ |
| 36. $\frac{x - 2}{6} = \frac{x}{2}$ | 37. $2(b - 3) = 3(b - 1)$ | 38. $8.3h - 2.2 = 6.1h - 8.8$ |

3-6 Ratios and Proportions

See pages
155–159.

Concept Summary

- A ratio is a comparison of two numbers by division.
- A proportion is an equation stating that two ratios are equal.
- A proportion can be solved by finding the cross products.

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example

Solve the proportion $\frac{8}{7} = \frac{a}{1.75}$.

$$\frac{8}{7} = \frac{a}{1.75} \quad \text{Original equation}$$

$$8(1.75) = 7(a) \quad \text{Find the cross products.}$$

$$14 = 7a \quad \text{Simplify.}$$

$$\frac{14}{7} = \frac{7a}{7} \quad \text{Divide each side by 7.}$$

$$2 = a \quad \text{Simplify.}$$

Exercises Solve each proportion. See Example 3 on page 156.

- | | | |
|------------------------------------|-------------------------------------|-------------------------------------|
| 39. $\frac{6}{15} = \frac{n}{45}$ | 40. $\frac{x}{11} = \frac{35}{55}$ | 41. $\frac{12}{d} = \frac{20}{15}$ |
| 42. $\frac{14}{20} = \frac{21}{m}$ | 43. $\frac{2}{3} = \frac{b + 5}{9}$ | 44. $\frac{6}{8} = \frac{9}{s - 4}$ |



3-7 Percent of ChangeSee pages
160–164.**Concept Summary**

- The proportion $\frac{\text{amount of change}}{\text{original amount}} = \frac{r}{100}$ is used to find percents of change.

Example

Find the percent of change. original: \$120
new: \$114

First, subtract to find the amount of change.

$$\$120 - \$114 = \$6 \quad \text{Note that since the new amount is less than the original, the percent of change will be a percent of decrease.}$$

Then find the percent using the original number, 120, as the base.

$$\begin{array}{l} \text{change} \rightarrow \frac{6}{120} = \frac{r}{100} \\ \text{original amount} \rightarrow \end{array} \quad \text{Percent proportion}$$

$$6(100) = 120(r) \quad \text{Find the cross products.}$$

$$600 = 120r \quad \text{Simplify.}$$

$$\frac{600}{120} = \frac{120r}{120} \quad \text{Divide each side by 120.}$$

$$5 = r \quad \text{Simplify.}$$

The percent of decrease is 5%.

Exercises State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent. See Example 1 on page 160.

45. original: 40
new: 32

46. original: 50
new: 88

47. original: 35
new: 37.1

48. Find the total price of a book that costs \$14.95 plus 6.25% sales tax.
See Example 3 on page 161.

49. A T-shirt priced at \$12.99 is on sale for 20% off. What is the discounted price?
See Example 4 on page 161.

3-8 Solving Equations and FormulasSee pages
166–170.**Concept Summary**

- For equations with more than one variable, you can solve for one of the variables by using the same steps as solving equations with one variable.

Example

Solve $\frac{x+y}{b} = c$ for x .

$$\frac{x+y}{b} = c \quad \text{Original equation}$$

$$b\left(\frac{x+y}{b}\right) = b(c) \quad \text{Multiply each side by } b.$$

$$x+y = bc \quad \text{Simplify.}$$

$$x+y-y = bc-y \quad \text{Subtract } y \text{ from each side.}$$

$$x = bc-y \quad \text{Simplify.}$$

- Extra Practice, see pages 825–828.
- Mixed Problem Solving, see page 855.

Exercises Solve each equation or formula for the variable specified.

See Examples 1 and 2 on pages 166 and 167.

50. $5x = y$, for x
51. $ay - b = c$, for y
52. $yx - a = cx$, for x
53. $\frac{2y - a}{3} = \frac{a + 3b}{4}$, for y

3-9 Weighted Averages

See pages
171–177.

Concept Summary

- The weighted average of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights.
- The formula $d = rt$ is used to solve uniform motion problems.

Example

SCIENCE Mai Lin has a 35 milliliters of 30% solution of copper sulfate. How much of a 20% solution of copper sulfate should she add to obtain a 22% solution?

Let x = amount of 20% solution to be added. Make a table.

	Amount of Solution (mL)	Amount of Copper Sulfate
30% Solution	35	$0.30(35)$
20% Solution	x	$0.20x$
22% Solution	$35 + x$	$0.22(35 + x)$

$$0.30(35) + 0.20x = 0.22(35 + x) \quad \text{Write and solve an equation.}$$

$$10.5 + 0.20x = 7.7 + 0.22x \quad \text{Distributive Property}$$

$$10.5 + 0.20x - 0.20x = 7.7 + 0.22x - 0.20x \quad \text{Subtract } 0.20x \text{ from each side.}$$

$$10.5 = 7.7 + 0.02x \quad \text{Simplify.}$$

$$10.5 - 7.7 = 7.7 + 0.02x - 7.7 \quad \text{Subtract } 7.7 \text{ from each side.}$$

$$2.8 = 0.02x \quad \text{Simplify.}$$

$$\frac{2.8}{0.02} = \frac{0.02x}{0.02} \quad \text{Divide each side by } 0.02.$$

$$140 = x \quad \text{Simplify.}$$

Mai Lin should add 140 milliliters of the 20% solution.

Exercises

54. **COFFEE** Ms. Anthony wants to create a special blend using two coffees, one priced at \$8.40 per pound and the other at \$7.28 per pound. How many pounds of the \$7.28 coffee should she mix with 9 pounds of the \$8.40 coffee to sell the mixture for \$7.95 per pound? *See Example 1 on page 171.*
55. **TRAVEL** Two airplanes leave Dallas at the same time and fly in opposite directions. One airplane travels 80 miles per hour faster than the other. After three hours, they are 2940 miles apart. What is the speed of each airplane? *See Example 3 on pages 172 and 173.*

Vocabulary and Concepts

Choose the correct term to complete each sentence.

- The study of numbers and the relationships between them is called (*consecutive, number*) theory.
- An equation that is true for (*every, only one*) value of the variable is called an identity.
- When a new number is (*greater than, less than*) the original number, the percent of change is called a percent of increase.

Skills and Applications

Translate each sentence into an equation.

- The sum of twice x and three times y is equal to thirteen.
- Two thirds of a number is negative eight fifths.

Solve each equation. Then check your solution.

- | | | |
|---|---------------------------|------------------------------------|
| 6. $-15 + k = 8$ | 7. $-1.2x = 7.2$ | 8. $k - 16 = -21$ |
| 9. $\frac{t-7}{4} = 11$ | 10. $\frac{3}{4}y = -27$ | 11. $-12 = 7 - \frac{y}{3}$ |
| 12. $t - (-3.4) = -5.3$ | 13. $-3(x + 5) = 8x + 18$ | 14. $5a = 125$ |
| 15. $\frac{r}{5} - 3 = \frac{2r}{5} + 16$ | 16. $0.1r = 19$ | 17. $-\frac{2}{3}z = -\frac{4}{9}$ |
| 18. $-w + 11 = 4.6$ | 19. $2p + 1 = 5p - 11$ | 20. $25 - 7w = 46$ |

Solve each proportion.

- | | | |
|-----------------------------------|-------------------------------------|-------------------------------------|
| 21. $\frac{36}{t} = \frac{9}{11}$ | 22. $\frac{n}{4} = \frac{3.25}{52}$ | 23. $\frac{5}{12} = \frac{10}{x-1}$ |
|-----------------------------------|-------------------------------------|-------------------------------------|

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.

- | | |
|----------------------------|-----------------------------|
| 24. original: 45
new: 9 | 25. original: 12
new: 20 |
|----------------------------|-----------------------------|

Solve each equation or formula for the variable specified.

- $h = at - 0.25vt^2$, for a
- $a(y + 1) = b$, for y
- SALES** Suppose the Central Perk coffee shop sells a cup of espresso for \$2.00 and a cup of cappuccino for \$2.50. On Friday, Destiny sold 30 more cups of cappuccino than espresso for a total of \$178.50 worth of espresso and cappuccino. How many cups of each were sold?
- BOATING** The *Yankee Clipper* leaves the pier at 9:00 A.M. at 8 knots (nautical miles per hour). A half hour later, *The River Rover* leaves the same pier in the same direction traveling at 10 knots. At what time will *The River Rover* overtake *The Yankee Clipper*?
- STANDARDIZED TEST PRACTICE** If $\frac{4}{5}$ of $\frac{3}{4} = \frac{2}{5}$ of $\frac{x}{4}$, find the value of x .

(A) 12

(B) 6

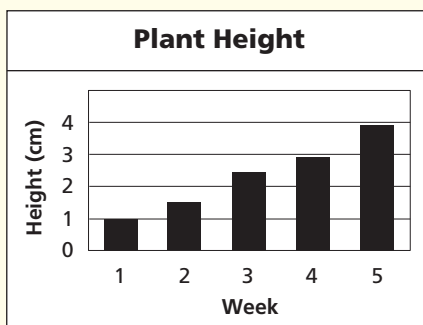
(C) 3

(D) $\frac{3}{2}$ 

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

- Bailey planted a rectangular garden that is 6 feet wide by 15 feet long. What is the perimeter of the garden? (Prerequisite Skill)
 - (A) 21 ft
 - (B) 27 ft
 - (C) 42 ft
 - (D) 90 ft
- Which of the following is true about 65 percent of 20? (Prerequisite Skill)
 - (A) It is greater than 20.
 - (B) It is less than 10.
 - (C) It is less than 20.
 - (D) Can't tell from the information given
- For a science project, Kelsey measured the height of a plant grown from seed. She made the bar graph below to show the height of the plant at the end of each week. Which is the most reasonable estimate of the plant's height at the end of the sixth week? (Lesson 1-8)



- (A) 2 to 3.5 cm
 - (B) 4 to 5.5 cm
 - (C) 6 to 7 cm
 - (D) 8 to 8.5 cm
- WEAT predicted a 25% chance of snow. WFOR said the chance was 1 in 4. Myweather.com showed the chance of snow as $\frac{1}{5}$, and Allweather.com listed the chance as 0.3. Which forecast predicted the greatest chance of snow? (Lesson 2-7)
 - (A) WEAT
 - (B) WFOR
 - (C) Myweather.com
 - (D) Allweather.com

- Amber owns a business that transfers photos to CD-ROMs. She charges her customers \$24.95 for each CD-ROM. Her expenses include \$575 for equipment and \$0.80 for each blank CD-ROM. Which of these equations could be used to calculate her profit p for creating n CD-ROMs? (Lesson 3-1)

- (A) $p = (24.95 - 0.8)n - 575$
- (B) $p = (24.95 + 0.8)n + 575$
- (C) $p = 24.95n - 574.2$
- (D) $p = 24.95n + 575$

- Which of the following equations has the same solution as $8(x + 2) = 12$? (Lesson 3-4)

- (A) $8x + 2 = 12$
- (B) $x + 2 = 4$
- (C) $8x = 10$
- (D) $2x + 4 = 3$

- Eduardo is buying pizza toppings for a birthday party. His recipe uses 8 ounces of shredded cheese for 6 servings. How many ounces of cheese are needed for 27 servings? (Lesson 3-6)

- (A) 27
- (B) 32
- (C) 36
- (D) 162

- The sum of x and $\frac{1}{y}$ is 0, and y does not equal 0. Which of the following is true? (Lesson 3-8)

- (A) $x = -y$
- (B) $\frac{x}{y} = 0$
- (C) $x = 1 - y$
- (D) $x = -\frac{1}{y}$

The Princeton Review

Test-Taking Tip

Questions 2, 6, 8 Always read every answer choice, particularly in questions that ask, "Which of the following is true?"

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

- Let $x = 2$ and $y = -3$. Find the value of $\frac{x(xy + 5)}{4}$. (Lesson 1-2)
- Use the formula $F = \frac{9}{5}C + 32$ to convert temperatures from Celsius (C) to Fahrenheit (F). If it is -5° Celsius, what is the temperature in degrees Fahrenheit? (Lesson 2-3)
- Darnell keeps his cotton socks folded in pairs in his drawer. Five pairs are black, 2 pairs are navy, and 1 pair is brown. In the dark, he pulls out one pair at random. What are the odds that it is black? (Lesson 2-6)
- The sum of the ages of the Kruger sisters is 39. Their ages can be represented as three consecutive integers. What is the age of the middle sister? (Lesson 3-4)
- On a car trip, Tyson drove 65 miles more than half the number of miles Pete drove. Together they drove 500 miles. How many miles did Tyson drive? (Lesson 3-4)
- Solve $7(x + 2) + 4(2x - 3) = 47$ for x . (Lesson 3-5)
- A bookshop sells used hardcover books with a 45% discount. The price of a book was \$22.95 when it was new. What is the discounted price for that book? (Lesson 3-7)

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- the quantity in Column A is greater,
- the quantity in Column B is greater,
- the two quantities are equal, or
- the relationship cannot be determined from the information given.

Column A	Column B
----------	----------

16.

$ a $	$ -a $
-------	--------

(Lesson 2-1)

17.

solution of $3x + 7 = 10$	solution of $4y - 2 = 6$
------------------------------	-----------------------------

(Lesson 3-4)

18.

the percent of increase from \$75 to \$100	the percent of increase from \$150 to \$200
--	---

(Lesson 3-7)

Part 4 Open Ended

Record your answers on a sheet of paper.

- Kirby's pickup truck travels at a rate of 6 miles every 10 minutes. Nola's SUV travels at a rate of 15 miles every 25 minutes. The speed limit on this street is 40 mph. Is either vehicle or are both vehicles exceeding the speed limit? Explain. (Lesson 3-6)
- A chemist has one solution of citric acid that is 20% acid and another solution of citric acid that is 80% acid. She plans to mix these solutions together to make 200 liters of a solution that is 50% acid. (Lesson 3-9)
 - Complete the table to show the liters of 20% and 80% solutions that will be used to make the 50% solution. Use x to represent the number of liters of the 80% solution that will be used to make the 50% solution.

	Liters of Solution	Liters of Acid
20% Solution		
80% Solution	x	
50% Solution	200	$0.50(200)$

- Write an equation that represents the number of liters of acid in the solution.
- How many liters of the 20% solution and how many of the 80% solution will the chemist need to mix together to make 200 liters of a 50% solution?